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This programed text on electromechanical energy conversion (motors and generators) was developed under contract with the U.S. Office of Education as Number 12 in a series of materials for use in an electrical engineering sequence. It is intended to be used in conjunction with other materials and with other short texts in the series. (DH)

No. 12

ELECTROMECHANICAL ENERGY CONVERSION

by

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Contract No. OE 4-10-102
U.S. Office of Education

This programmed text on the subject of electric motors and generators covers enough of the fundamental principles to enable you to answer questions of "how" and "why" that arise in connection with their use in practical applications. Important constructional features are explained and related to the fundamental laws of electromagnetism. Study of this text will enable you to do such practical things as recognize the difference between a-c and d-c machines, relate operating voltage to physical dimensions and speed, and derive equivalent circuits.

Both motors and generators operate as a result of a single physical principle, which you have learned as the Lorentz force law. This law relates the force on a moving charge to its velocity and the strength of the magnetic field in which it is moving, according to the formula $\vec{F} = q(\vec{w} \times \vec{B})$ which is discussed on the next page.

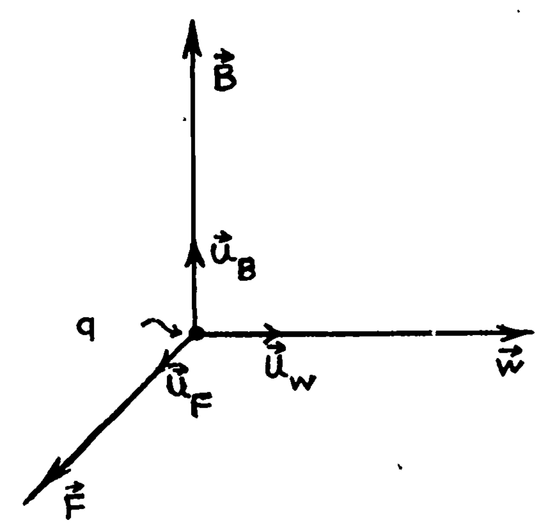

Figure 1.

LORENTZ FORCES

In Fig. 1 the dot represents a charge q, moving with velocity w in a magnetic field B. It experiences the Lorentz force

$$\vec{F} = q(\vec{w} \times \vec{B})$$

In the application of this equation to motors and generators, vectors $\vec{w}$ and $\vec{B}$ will be perpendicular to each other. They can be written

$$\vec{w} = w\,\vec{u}_W \qquad \text{and} \qquad \vec{B} = B\,\vec{u}_B$$

where w and B are scalar numbers, which can be positive or negative, and $\vec{u}_W$ and $\vec{u}_B$ are perpendicular unit vectors, as in Fig. 1. Let $\vec{u}_F$ be a unit vector given by

$$\vec{u}_F = \vec{u}_W \times \vec{u}_B$$

as illustrated in Fig. 1. In terms of this, the force vector can be written

$$\vec{F} = (\qquad\qquad)\vec{u}_F = F\,\vec{u}_F$$

In MKSC units the quantities in the above equations are:

w __________, F __________, B __________, q __________

<u>Answers:</u>

$\vec{F} = (qwB)\vec{u}_F$

w in meters/sec.

B in webers/sq. meter

F in newtons

q in coulombs.

In the space above, draw vectors of $\vec{w}$, $\vec{B}$, and $\vec{F}$ when w is negative and B is positive, using unit vectors $\vec{u}_w$ and $\vec{u}_B$ shown in Fig. 2.

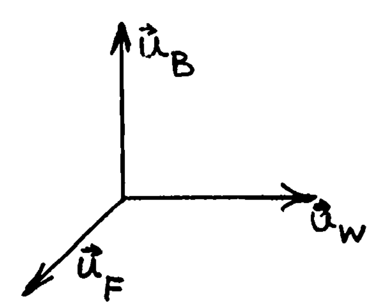

Figure 2.

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Recall that $\vec{u}_W$ and $\vec{u}_B$ are used to express $\vec{w}$ and $\vec{B}$ as follows:

$$\vec{w} = w(\quad) \qquad \text{and} \qquad \vec{B} = B(\quad)$$

Now suppose w is a negative number and B is positive. Draw the actual $\vec{w}$, $\vec{B}$ and $\vec{F}$ vectors in the space above Fig. 2.

In view of this example, it may be said that $\vec{F}$ is in the direction of $\vec{u}_F$ whenever the algebraic sign of the product qwB is __________ .

Furthermore, in words, the direction of $\vec{u}_F$ is the direction of advance of a right-hand screw if it is rotated ____ .

<u>Answers:</u>

$\vec{W} = W(\hat{u}_W)$ $\vec{B} = B(\hat{u}_B)$

positive

from $\hat{u}_W$ to $\hat{u}_B$ through the smaller angle.

(Note: if you neglected to say "through the smaller angle", observe that there are two ways to rotate from $\hat{u}_W$ to $\hat{u}_B$.)

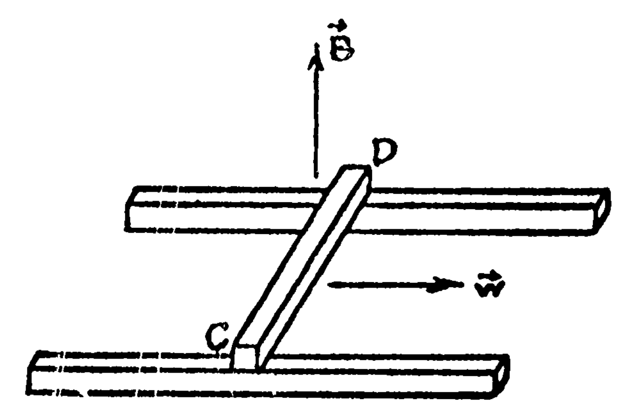

Figure 3.

If the charge q of Figs. 1 and 2 is a free electron in a conducting rod sliding along a pair of rails, as in Fig. 3, and if w and B are positive numbers, since the change on an electron is negative, it will experience a force

toward end ________ (C or D)

In certain materials, like semiconductors, positive charges move also. Under the conditions of Fig. 3, positive charges would move

toward end ________ (C or D)

Which of the following statements applies?

(1) If electrons are the only moving charges, end D becomes negative, and there will be no charge at end C. If both positive and negative charges move, end D will become negative and end C will become positive.

(2) In either of the cases described under (1), D will acquire negative charge and C will acquire positive charge.

Answers:

Toward end D

Toward end C

Statement 2 is true.

If you choose statement 1, you probably thought that C will remain uncharged if electrons are the only moveable charges. If so, you forgot that the removal of electrons from end C will leave it positively charged.

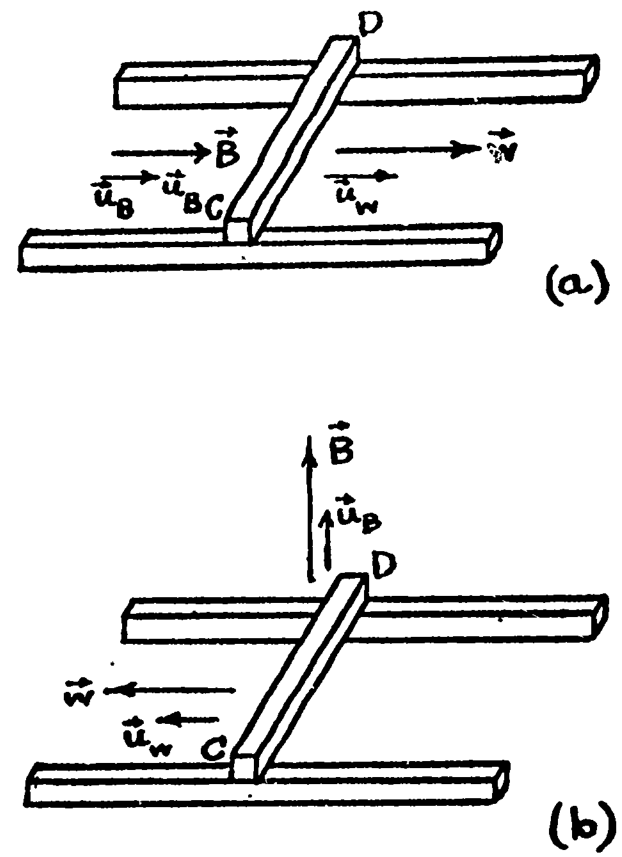

Figure 4.

In both parts of Fig. 4, assume $\vec{w}$ and $\vec{B}$ are actual directions. This means that in the relations $\vec{w} = w\,\vec{u}_w$ and $\vec{B} = B\,\vec{u}_B$,

w and B are both ______________

In Fig. 4a,

positive charges will ______________

negative charges will ______________

In Fig. 4b,

positive charges will ______________

negative charges will ______________

Again referring to Fig. 4b, if w remains the same, and if, in the relation,

$$\vec{B} = B\,\vec{u}_B$$

B is a negative number

end C will become __________ and end D will become __________.

Answers:

w and B are both positive

positve charges will not move.

negative charges will not move, (this is because $\vec{w}$ and $\vec{B}$ are colinear).

positive charges will go to end D.

negative charges will go to end C.

End C will become positive and end D will become negative.

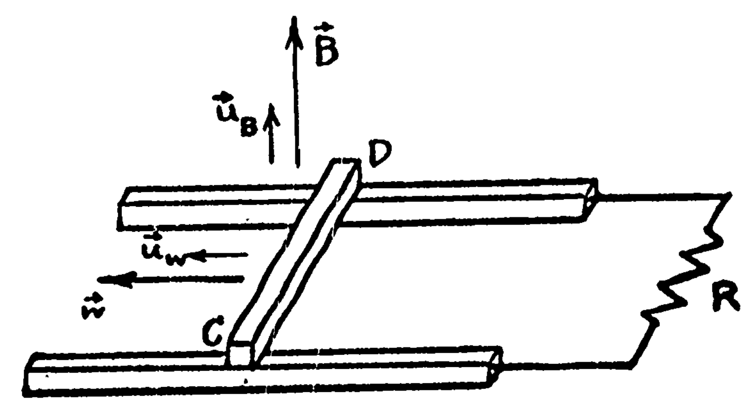

Figure 5.

Now suppose a current path is provided between the rails, as indicated by resistor R in Fig. 5. Directions are the same as in Fig. 4b, and as long as the bar is in motion, a current will flow around the completed circuit.

Show its direction by drawing an arrow next to the moving bar.

Does this arrow represent a vector? ____________

Explain your answer. ____

____

____

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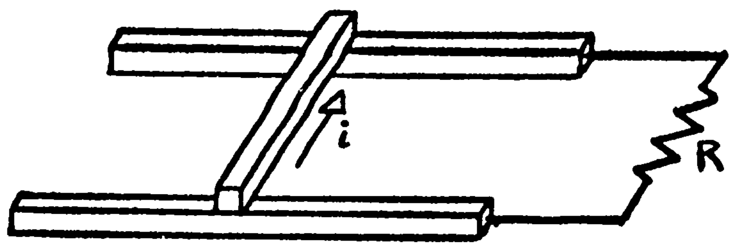
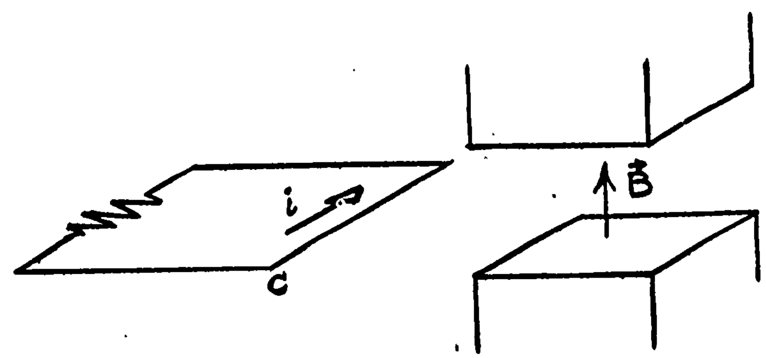
<u>Answers:</u>



No

This arrow represents the <u>reference</u> <u>direction</u> of the current, which is a <u>scalar</u>. For that reason, a different kind of arrowhead is shown, to make this distinction.

Having reviewed the theory of motion of charges caused by the Lorentz force, let us consider a review example. In the figure shown below, the loop of conducting wire moves through the magnetic field in the airgap between the poles of a magnet. On the set of axes plot the current i, showing its general nature as a function of distance, assuming that the loop moves at constant velocity.



If you have any difficulty with this problem, talk it over with your instructor.

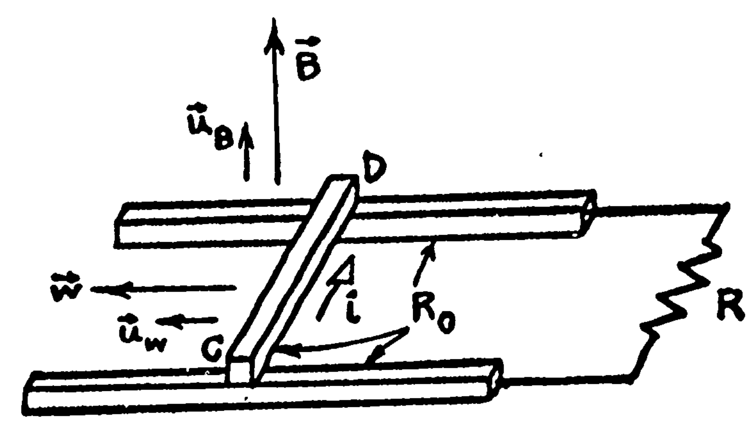


Figure 6.

ELECTROMOTIVE FORCE

In order to define electromotive force, we consider power flow in a closed circuit in which charge is being moved by the Lorentz force. In Fig. 6 assume w and B are constant and algebraically positive, and that the current is i. The rails between the bar and R will have electrical resistance, and so will the bar. Let R_0 be their combined resistance. With a current i flowing, the power dissipated in the circuit will be

$$(\quad)(\qquad\qquad)$$

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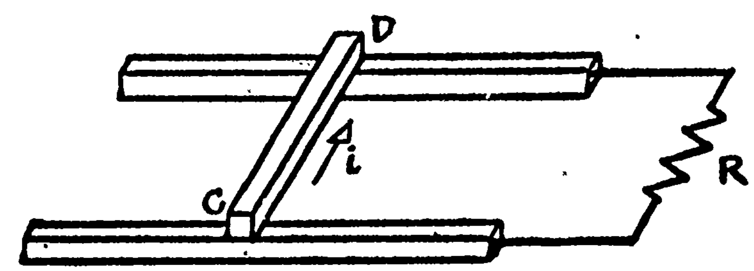
Answer:

$(i^2)(R + R_0)$



The previous frame established the expression

$$i^2(R + R_0)$$

for the power dissipated in the circuit. This is also the energy dissipated in <u>unit time</u>, (the rate of energy dissipation). This must be equal to the rate at which the Lorentz force does work.

Let the symbol e represent the work done (energy expended) per unit charge by the Lorentz force in moving **postive charge in the bar from C to D, or** alternately in moving unit negative charge from

_______ to __________

Also, the amount of charge moved per unit time will be ______________. Thus, the amount of work done by the Lorentz force per unit time (rate of expenditure of energy) will be

(   )(   )

<u>Answers:</u>

D to C

current i

(e)(i)

If you did not get ei without looking at the answer, go to page 19.

If you did get it, go to p. 21.

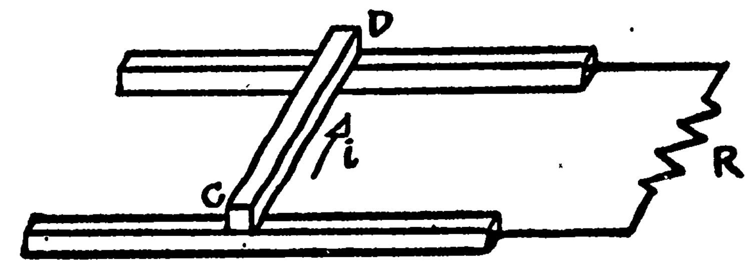
You probably did not properly interpret i as the amount of charge transferred from C to D in one second. If e is the amount of energy expended by the Lorentz force on one unit of charge (coulomb), the amount of work done on 2 coulombs would be $2e$, and the amount done on i coulombs (the amount of charge transferred in one sec.) will be ei joules. This is the work done in one second (the rate of doing work) or

__________ in units of ____________

20

<u>Answer.</u>

Power in watts.



As a final check, suppose negative charges are the only ones that move. In that case, e is the work done per unit negative charge in moving negative charge from

___ to ___

The amount of negative charge moved in this direction in one second will be

___ (i or $-i$)

Thus, the rate of doing work by the Lorentz force will be

___

Thus the rate of doing work in the case of positive charge motion and the rate of doing work in the case of negative charge motion are ______ __________.

<u>Answers:</u>

D to C

i

Because <u>negative</u> charges flowing <u>opposite</u> to the reference arrow results in a current <u>in the direction</u> of the arrow.

ei

the same.

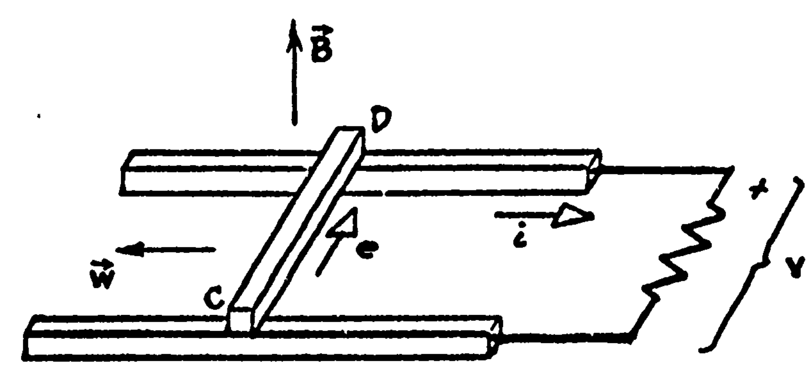


Figure 7.

Figure 7 is like figure 6, except that the current reference arrow has been moved, and in its place there is an arrow labeled e. This is the <u>reference direction</u> of the scalar quantity e, being the direction <u>positive</u> charge is moved when the Lorentz force is <u>doing</u> work. The quantity e is called the **<u>electromotive force</u>** acting in the direction of its reference arrow. It is abbreviated <u>emf</u>.

From conservation of energy, <u>the rate at which the Lorentz force does work</u> equals <u>the rate at which energy is dissipated</u>. Thus, using previous results, and equating these two, yields

$$\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$

Canceling the appropriate factors gives

$$\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$

Also, using v as a symbol for the terminal voltage, from Ohm's law we have

$$v =$$

and so we can also write

$$v = (\quad) - (\qquad).$$

Answers:

$ei = i^2(R + R_o)$

$e = iR + iR_o$

$v = iR$

$v = e - iR_o$

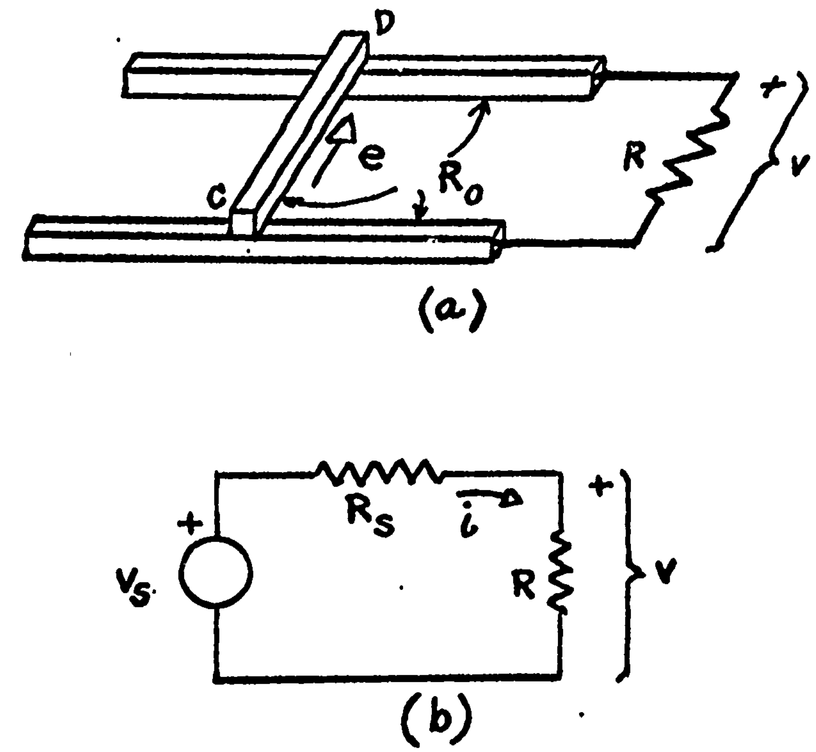


Figure 8.

Figure 8 is used to show how the previous result can be used to establish the <u>equivalent circuit</u> for this elementary "generator." For Fig. 8b, the Kirchhoff voltage equation is

$$v_s =$$

But $v = iR$ can be used in the above to give an equation which can be solved for v to give

$$v =$$

Thus, Fig. 8b can be used as an equivalent circuit if

$$v_s =$$

$$R_s =$$

Answers:

$v_s = i(R_s + R)$

$v = v_s - iR_s$

$v_s = e$

$R_s = R_o$

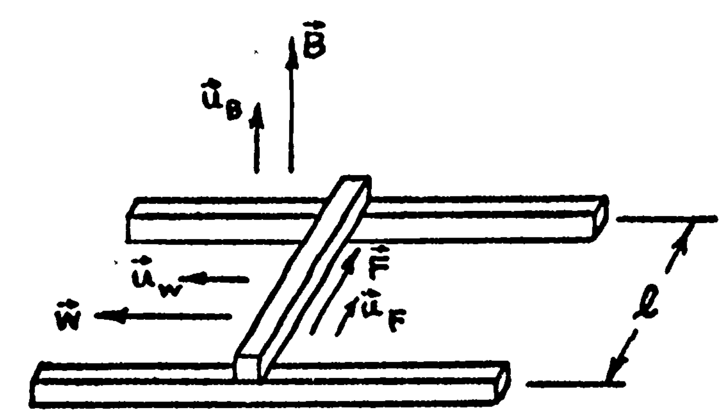


Figure 9.

An expression for e in terms of w, B, and conductor dimensions is yet to be considered. In terms of the vectors in Fig. 9, the Lorentz force on a <u>unit</u> positive charge is $\vec{F} = F\,\vec{u}_F$, and

$$F = \_\_\_\_\_\_$$

The work done in moving a unit positive charge the length (ℓ) of the bar from C to D is therefore

$$e = \_\_\_\_\_\_$$

Observe that <u>when the conductor is perpendicular to both</u> $\vec{w}$ and $\vec{B}$, as in Fig. 9, the reference arrow for e (not a vector) is in the direction of the unit vector

$$\vec{u}_F = \_\_\_\_\_\_\_\_ \text{ (in terms of } \vec{u}_W \text{ and } \vec{u}_B\text{)}$$

This perpendicular relationship is <u>always</u> present in motors and generators. See your reference text for a more general treatment of Motional Induced Voltage (Sec. 4-4).

Answers:

$F = wB$

$e = \ell wB$

$\vec{u}_F = \vec{u}_w \times \vec{u}_B$

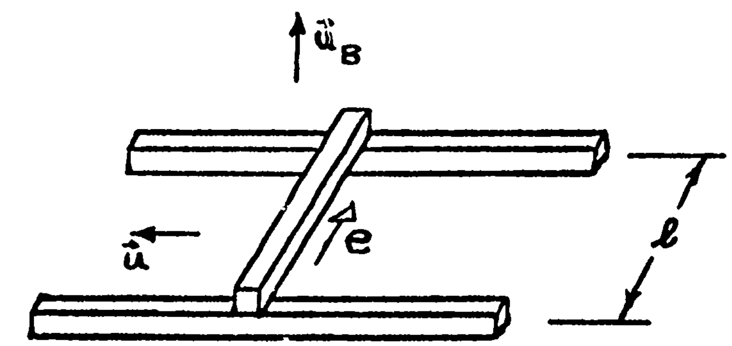


Figure 10.

<u>Review:</u>

Referring to Fig. 10, we have shown that e, the ______________________ __________, is given by

$$e = \ell w B$$

The quantity e is the work per unit charge done by the ___________ force, and is measured in ___________.

The reference direction for e is the same as

(1) The direction of $\vec{u}_W \times \vec{u}_B$

(2) The direction of $\vec{w} \times \vec{B}$

(3) Either (1) or (2); i.e. they are really the same.

Choose one.

Note: in this question, we are still using

$$\vec{w} = w\ \vec{u}_W$$

$$\vec{B} = B\ \vec{u}_B$$

<u>Answers:</u>

electromotive force

Lorentz volts

The answer to the multiple choice question is (1).

If you answered (1), go to p. 33.

If you answered (2) or (3), to to p. 31.

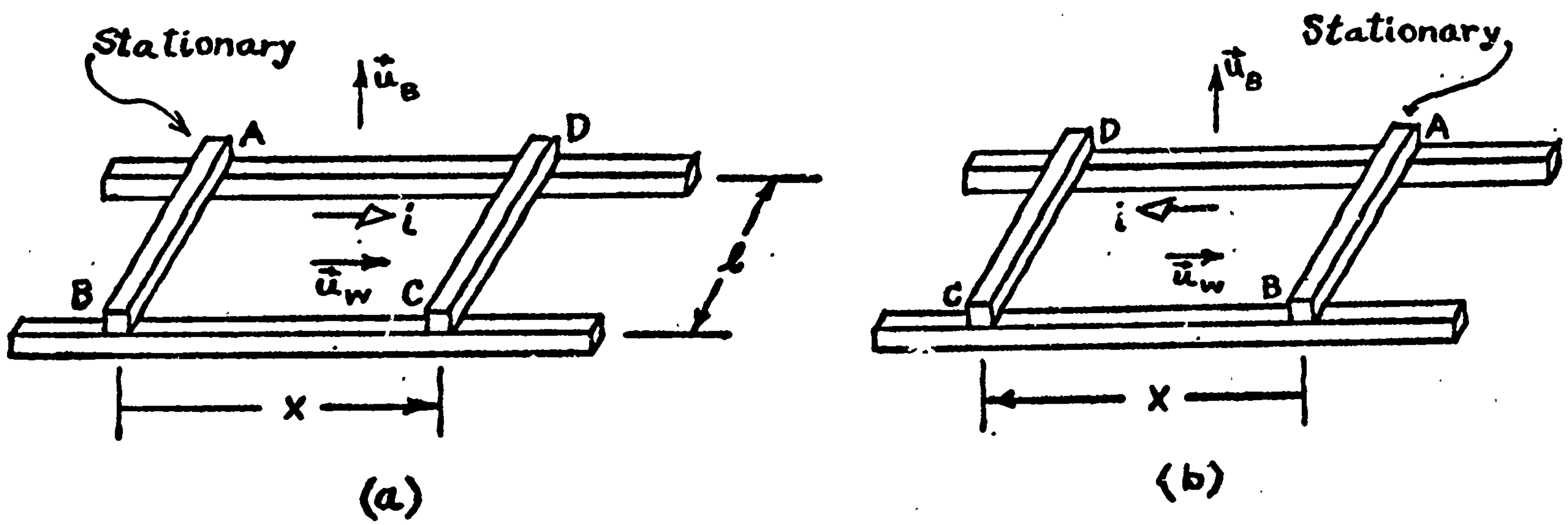
Observe that either w or B (in equations $\vec{w} = w\,\vec{u}_w$ and $\vec{B} = \vec{u}_B$) can be negative. This will change the <u>direction</u> of $\vec{w} \times \vec{B}$, and hence would change the reference direction of e. However, since

$$e = \ell wB$$

if w or B becomes negative, e will itself become negative. If the reference direction changes, the combination of a changed reference direction <u>and</u> a changed sign of e would mean that the actual electromotive force would be <u>unchanged</u>. This is contrary to physical facts.

If the <u>actual</u> direction of $\vec{w}$ or $\vec{B}$ is changed, the <u>actual</u> direction of e must change. Hence, $\vec{w} \times \vec{B}$ cannot determine the reference direction. The reference direction is determined by $\vec{u}_w \times \vec{u}_B$ which does not change with changing signs of w or B.

go to page 33.



Stationary
$\vec{u}_B$
A
D
i
$\vec{u}_W$
B
C
ℓ
X
(a)
Stationary
$\vec{u}_B$
D
A
i
$\vec{u}_W$
C
B
X
(b)

Figure 11.

At this point let us stop and refer to Fig. 11a. Bars AB and CD, and the rails are made of the same materials, having a resistance R_a ohms per meter. Bar AB is stationary, and bar CD is moving. Resistance at the contact points between bars and rails can be assumed to be zero.

(a) Show the reference arrow for e in bar CD

(b) Solve for e in terms of the rate of change of x (i.e. dx/dt)

(c) Obtain an equation for i in terms of x and dx/dt.

After you have done these for Fig. 11a, repeat the above three steps for Fig. 11b.

Note: The equation asked for in (c) will be approximate since it will not include the effect of the circuit inductance.

See your instructor, if there are any features of this problem you do not understand.

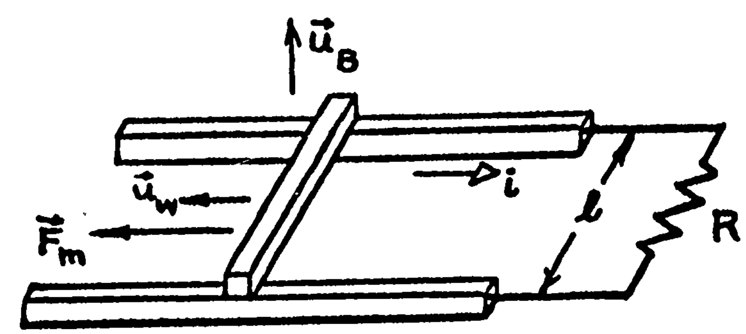


Figure 12.

The arrangement we have been considering, in which a conducting bar slides on a pair of rails, incorporates the essential features of a mechanical-to-electrical ENERGY CONVERSION device. We have seen that electrical energy is generated at the rate _____.

Of course, the bar will not slide by itself; an external mechanical force $\vec{F}_m$ must be applied to the bar, as in Fig. 12. We shall express this force in terms of the unit vector $\bar{u}_w$, thus

$$\vec{F}_m = F_m \vec{u}_w$$

defines the scalar value F_m. In your reference text it is shown that the Lorentz force on the moving charges (the current) results in the formula

$$F_m = \ell i B$$

for this particular configuration. (Note that $\vec{F}_m$ is the mechanical force <u>on</u> the bar, and thus is opposite to the force exerted <u>by</u> the bar).

The rate of doing mechanical work on the bar is ________ (in terms of w and F_m) or ____________ (in terms of ℓ, w, i, B).

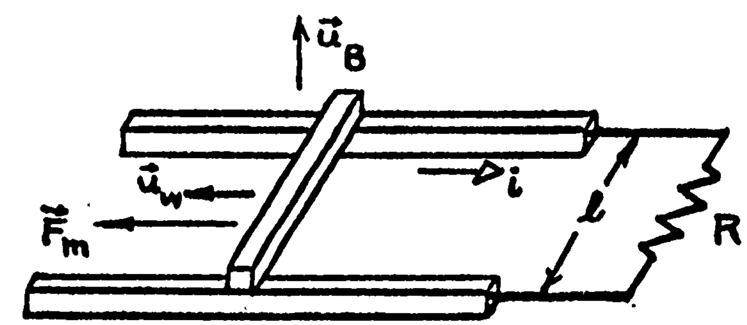
Answers:

ie

wF_m

$w\ell iB$

Observe that $\vec{F}_m$ is the resultant mechanical force, being the vector sum of the applied and frictional forces.



We have just found that the mechanical power is

$$P_m = w\ell iB$$

and that the electrical power is

$$P_e = ei$$

But, it is also true that

$e =$ ________ (draw the reference arrow for e on the figure on p. 36).

Thus, in terms of w, B, i, ℓ, the electrical power is also given by

$$P_e = \underline{\qquad\qquad}$$

It is thus seen that the mechanical power and the electrical power are ________ ________. In both formulas, the unit of power is ________.

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<u>Answers:</u>

$e = \ell w B$

$P_e = i\ell\, wB$

the same

watts.

Power has a direction of flow, and, being a scalar quantity, has a reference direction. For the conditions described in Fig. 12 (when w, B, i are all positive) mechanical power flows into the bar. Thus, the reference direction for mechanical power P_m is

____________ the bar

Also, under the same conditions, electrical power flows from the bar into the circuit elements R_O and R_1 where it appears as heat. Accordingly, the reference direction of P_e is

____________ the bar

40

Answers:

into

out of

It may be said that the bar is a region of space in which mechanical power is converted to electrical power.

ELEMENTARY ROTATING MACHINES

The sliding bar arrangement incorporates the essential elements of mechanical-electrical energy conversion, as in a generator. However it is not practical, particularly for the obvious reason that the bar cannot move indefinitely, since it will ultimately reach the end of the rails.

To construct a practical generator it is necessary that continuous motion shall be possible. This thought leads inevitably to the idea of continuous motion around a circle.

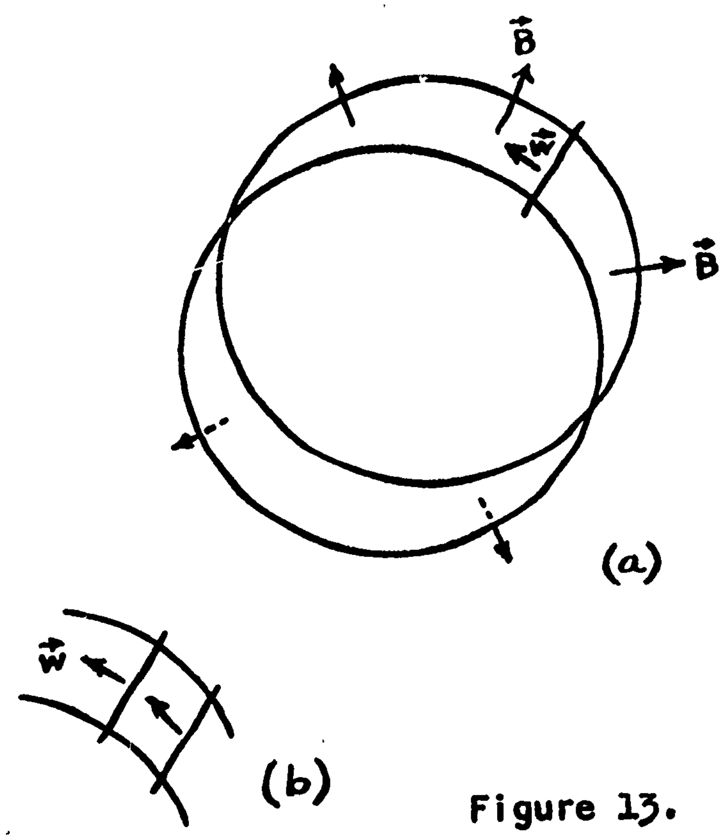


Figure 13.

One rather obvious way to overcome the difficulty of finite rails would be to bend them into circles, as shown in Fig. 13a. Then, if the $\vec{B}$ vector could everywhere radiate from the center, the conditions prescribed previously would prevail at each point on the periphery.

Such a $\vec{B}$ field arrangement can be attained, and so this is a feasible plan for a d-c generator. Certain practical difficulties, like how to provide the mechanical force on the bar, can be overcome. However, there is one serious limitation; the voltage attainable between the rails cannot exceed the emf (electromotive force) of the sliding bar. We might add a second bar, as in Fig. 13b. This will increase the capacity of the device to deliver current, by providing a second current path, but since the bars are in parallel, the voltage will not be changed.

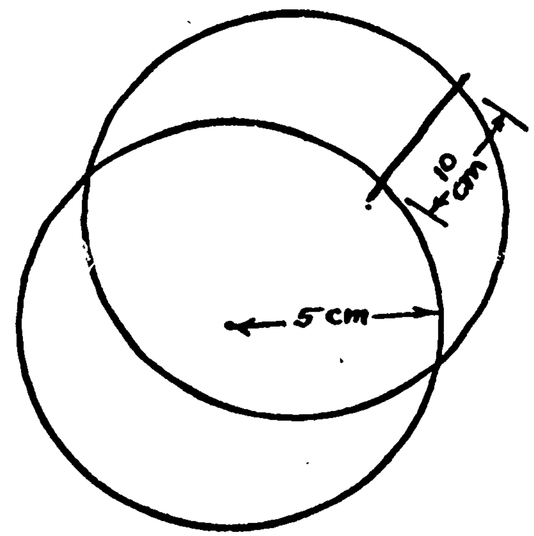


Figure 14.

This observation leads to the conclusion that there is a maximum voltage attainable from such a machine, for a given physical size and speed. For example, suppose you want to construct a physically small machine not more than 10 cm long and 10 cm in diameter, as in Fig. 14, with a rotational speed of 3600 rev. per min. (60 rev. per sec.).

Since the length of path traveled in one revolution is

$$(\quad)\pi \text{ or approximately } \_\_\_\_\_\_\_\_$$

the velocity magnitude is approximately

$$w = \_\_\_\_\_\_\_\_\_\_ \text{ meters per sec.}$$

Practical limitations on magnetic materials make it difficult to attain values of B much in excess of 1.5 webers/sq. meter. Using this value, the voltage attainable would be

$$(\quad)(\quad)(\quad) = \_\_\_\_\_\_\_ \text{ volts.}$$

<u>Answers:</u>

$(.1)\pi = .3$ meter (Approx.)

$w = 18$ meters/sec.

$(.1)(1.5)(18) = 2.7$ volts.

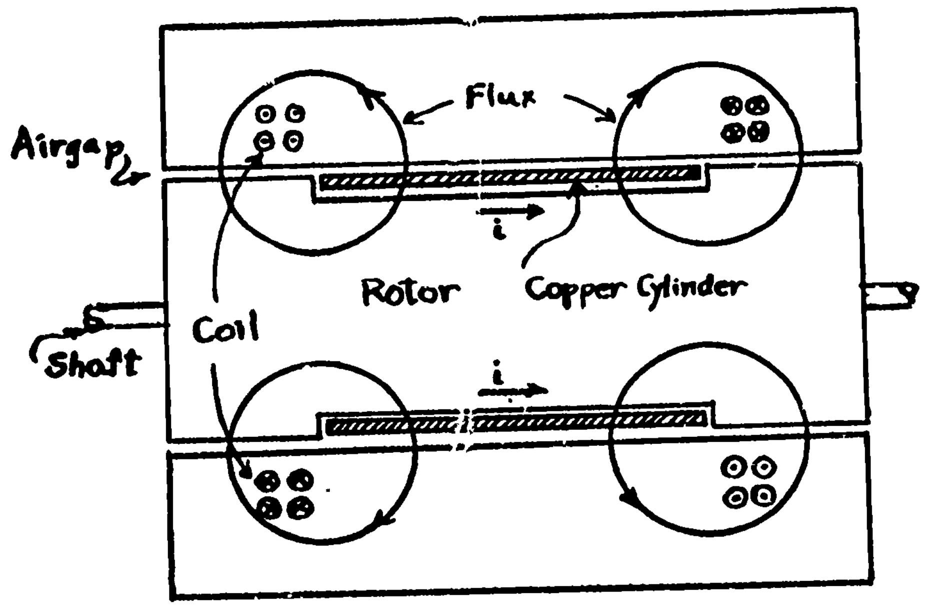


Figure 15.

Generators of the type just considered are commercially available, having piactical applications where high current and low voltage is required, as in electro-plating and other electro-chemical applications.

In practice, instead of having a single conductor move over circular rails, a cylindrical conductor is rotated in the manner indicated by the cross sectional view in Fig. 15a. The radial magnetic field is provided by the two coils indicated. When the cylinder is connected to an external circuit by liquid metal moving contacts at its edges (not shown in the figure) current will flow as indicated.

This type of machine is called an <u>acyclic</u> generator, the word "acyclic" meaning that there is no repeating cycle as the rotor rotates; the voltage and current are independent of the position of the rotor. Ratings of 150,000 amperes at 45 volts are typical.

Suppose the length and diameter of t lindrical conductor are equal, and that B and rotational speed are the sam in the example on p. 45, for which we found that 10 cm. by 10 cm dimensions would yield 2.7 volts. What dimensions are required to obtain 45 volts?

Answer:

Approximately 40 cm.

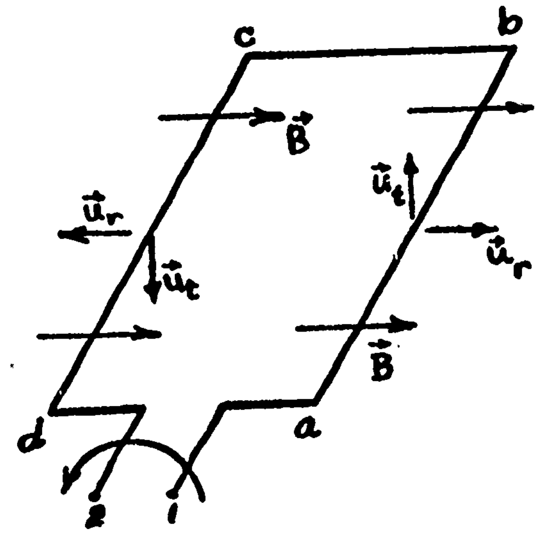
The electromotive force is proportional to the product of length and peripheral velocity. But peripheral velocity is proportional to radius (and hence proportional to length, since the radius is $\ell/2$). Thus, since length and diameter are equal, emf is proportional to ℓ^2, giving

$$\frac{45}{2.7} = \frac{\ell^2}{(10)^2}$$

or

$$\ell = 10\sqrt{\frac{45}{2.7}} = 10 \times \sqrt{16} = 40 \text{ cm}$$

approximately.



Each $\vec{u}_t$ is tangential to the circle of motion.

Figure 16.

We shall now consider the more typical type of machine, beginning with an example where <u>two</u> conductors are connected in series, so that their emf's add. In Fig. 16 you should focus your attention on loop sides a-b and c-d which are the two conductors in question.

Unit vectors $\vec{u}_t$ (tangential) and $\vec{u}_r$ (radial) are shown at each conductor. Due to rotation, side a-b is moving upward at the instant pictured, and side c-d is moving downward. Thus, in the formula

$$\vec{w} = w\,\vec{u}_t$$

w is $\underset{\text{(sign ?)}}{\underline{\hspace{3cm}}}$ in each case.

Unit vector $\vec{u}_r$ is radial, pointing away from the axis of rotation at each conductor. Since the actual $\vec{B}$ is uniform, and to the right, in the equation

$$\vec{B} = B\,\vec{u}_r$$

B is $\underset{\text{(sign ?)}}{\underline{\hspace{3cm}}}$ for conductor a-b and $\underset{\text{(sign ?)}}{\underline{\hspace{3cm}}}$ for conductor c-d.

Answers:

positive

positive negative

Note: If you wonder why $\vec{u}_t$ and $\vec{u}_r$ are now used in place of the former notation $\vec{u}_w$ and $\vec{u}_B$, this is done because these vectors change position when the loop rotates. The new subscripts (t for tangential and r for radial) imply that the properties of these vectors are invariant with respect to the loop.

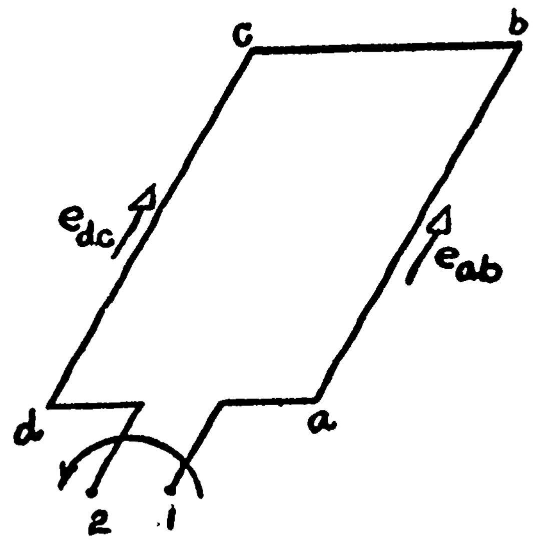


Figure 17.

To carry this thought about signs a bit further, let B_m be the <u>magnitude</u> of $\vec{B}$ at each conductor. Then, at a-b

$$\vec{B} = (\qquad)\vec{u}_r$$

and at conductor c-d,

$$\vec{B} = (\qquad)\vec{u}_r$$

Note that $\vec{u}_r$ is different in these two expressions, but that in each case it is ____________________

Earlier, we found that there is a specific reference direction for e when it is expressed by ℓwB. Reference directions for the two e's are shown in Fig. 17. Each of them is related to $\vec{u}_t$ and $\vec{u}_r$ (previously respectively $\vec{u}_W$ and $\vec{u}_B$) by the rule ____

____

Draw $\vec{u}_t$ and $\vec{u}_r$ at each conductor in Fig. 17, to convince yourself that the same rule applies to both.

<u>Answers:</u>

at a-b $\vec{B} = (B_m)\vec{u}_r$

at c-d $\vec{B} = (-B_m)\vec{u}_r$

directed radially outward.

The direction of advance of a right-hand screw when rotated through the smaller angle from $\vec{u}_t$ to $\vec{u}_r$.

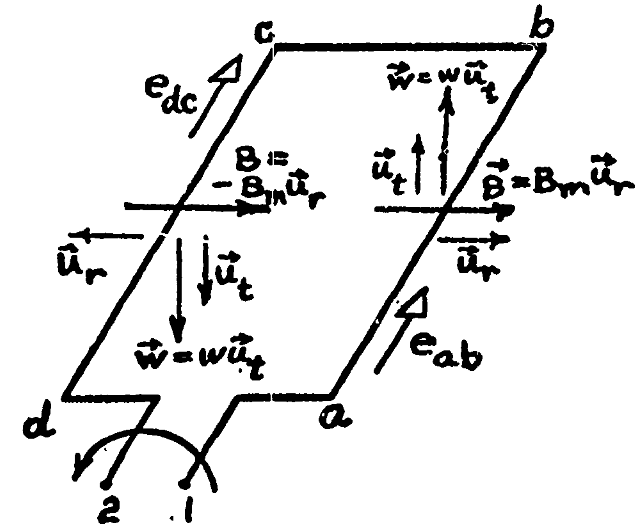


Figure 18.

Now let us apply the formula ℓwB to each conductor, observing that w is positive and the same in both cases. We get:

$$e_{ab} = \_\_\_\_\_\_\_\_\_\_ \quad \text{and} \quad e_{dc} = \_\_\_\_\_\_\_\_\_\_$$

The entire emf of the loop, which we shall call e_{12} is the work done by the Lorentz force in moving a unit positive charge from 1 to 2 (that is, from 1 to a to b to c to d to 2). The work from 1 to a, b to c, and d to 2 is ________.

Thus,

$$e_{12} = \text{work from } \_\_\_ \text{ to } \_\_\_ + \text{ work from } \_\_\_ \text{ to } \_\_\_$$

But, the work from c to d is _________. Thus

$$e_{12} = (\qquad) + (\qquad)$$

and, finally, in terms of ℓ, w, and B_m,

$$e_{12} = \_\_\_\_\_\_\_\_\_\_.$$

Answers:

$e_{ab} = \ell w B_m$, $e_{dc} = \ell w(-B_m)$

or $-\ell w B_m$

zero, because in these conductor segements the Lorentz force is <u>normal</u> to the conductor. Be sure you see why.

work from a to b + work from c to d

$-e_{dc}$

$e_{12} = (e_{ab}) + (-e_{dc})$

$e_{12} = 2\ell w B_m$

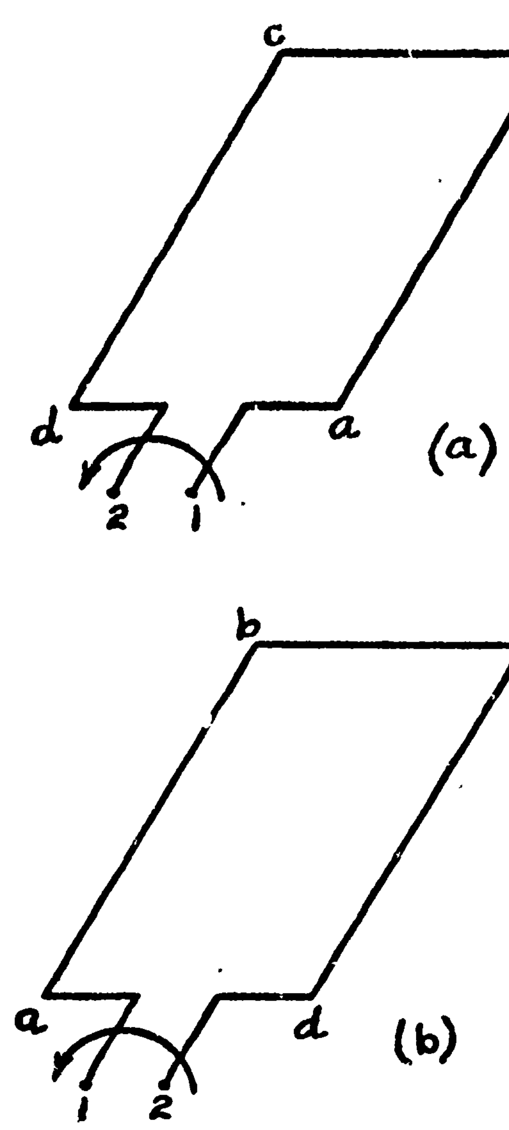


Figure 19

Thus, we see that the voltage at the terminals is twice the emf of one loop side. To make sure you did not get lost in the negative signs, place arrows on Fig. 19a to show the <u>actual</u> directions of the emf's in the two loop sides (as distinct from the reference directions shown in Fig. 18).

In Fig. 19b the loop has been rotated through 180°. Show the <u>actual</u> directions of the emf's in this case also. For Fig. 19a we found that

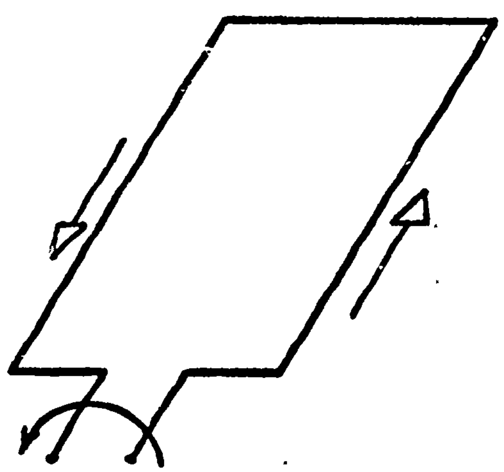
$$e_{12} = 2\ell w B_m$$

The corresponding expression for Fig. 19b is

$$e_{12} = \underline{\qquad\qquad}$$

56

Answer:



$-2\ell wB_m$

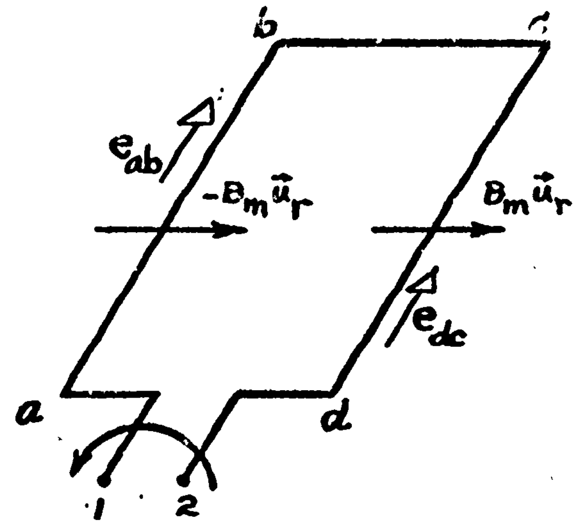


Figure 20.

Compared to the <u>acyclic</u> type of construction, it is seen that we have "paid a price" for being able to add the emf's of two conductors. That is, when the loop reverses its position the sign of the emf ________________.

This is so important that it is worthwhile to show that we get the same result using the reference directions shown in Fig. 20. In the new position, shown in that figure, e_{12} is related to e_{ab} and e_{dc} in the same way as before, namely

$$e_{12} = (\quad) - (\quad)$$

However, in this case

$$e_{ab} = \underline{\qquad\qquad} \quad \text{and} \quad e_{dc} = \underline{\qquad\qquad}$$

and so

$$e_{12} = \underline{\qquad\qquad}$$

2

<u>**Answers:**</u>

also reverses.

$e_{12} = e_{ab} - e_{dc}$

$e_{ab} = -\ell w B_m$ and $e_{dc} = \ell w B_m$

$e_{12} = -2\ell w B_m$

Note the change in sign!

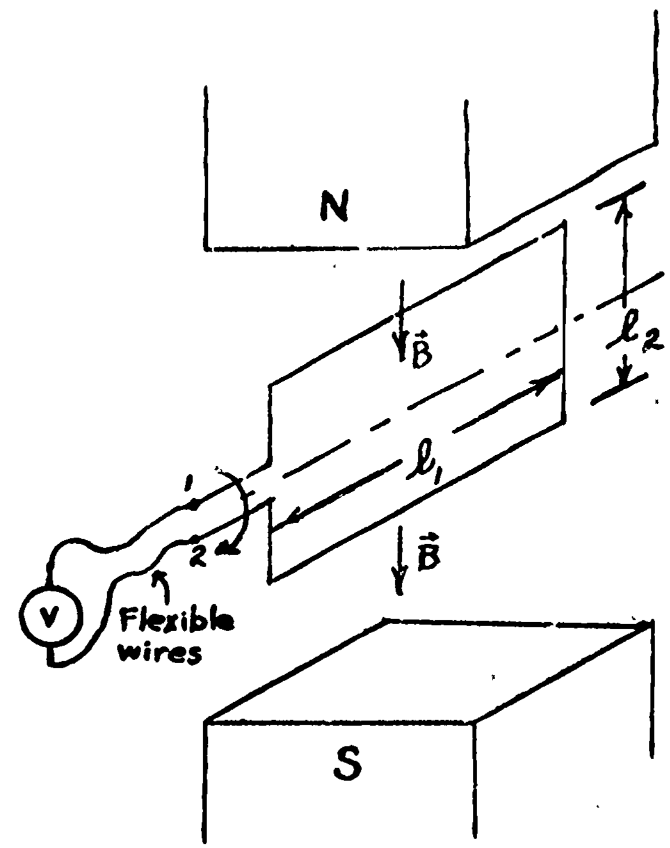


Figure 21.

This is a review problem. In Fig. 21 a pair of magnetic poles labeled N and S creates a uniform vertical flux in the region of a rotating loop. The loop rotates clockwise when viewed from the end having terminals 1, 2. The symbol (V) indicates an oscilloscope to be used for measuring voltage. At the instant shown in Fig. 21, the magnitude of B is 0.6 webers/sq. meter, ℓ is 5 cm, ℓ_2 is 4 cm and rotation is at 800 rev./min. What will be the instantaneous voltage as indicated by the oscilloscope (including sign)?

After another 1/2 revolution, what will be the instantaneous indication of the oscilloscope?

What will these answers be if $\ell_1 = 4$ cm and $\ell_2 = 5$ cm?

The answer is approximately a power of ten. What is it?

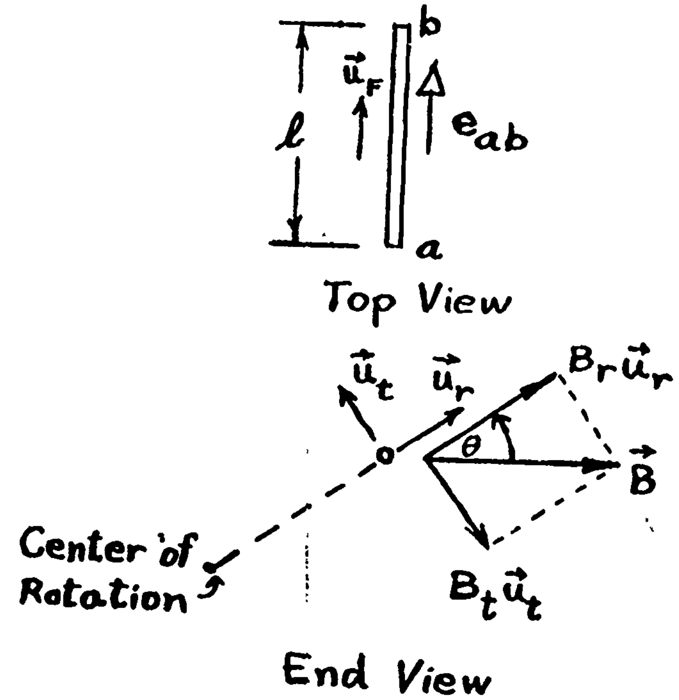


Figure 22.

No mention has been made about the emf when the $\vec{B}$ vector is not parallel to the plane of the loop. Such a case is shown in Fig. 22. Observe that $\vec{B}$ can be written as the sum of two components, as indicated. The $B_t\ \vec{u}_t$ component produces no emf because it is colinear with ______________.

Thus,

$$e_{ab} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Also, B_r can be obtained from $\vec{u}_r$ and $\vec{B}$ as follows

$$B_r = (\qquad) \cdot (\qquad)$$

and in this case, if B_m is the magnitude of $\vec{B}$,

$$B_r = \_\_\_\_\_\_\_\_\_\_\_\_ .$$

Answers:

$\vec{w}$ (or $w\,\vec{u}_t$)

That is, $B_t\vec{u}_t \times \vec{w} = 0$

$e_{ab} = \ell w B_r$

$B_r = \vec{B} \cdot \vec{u}_r$

$B_r = B_m \cos\theta$

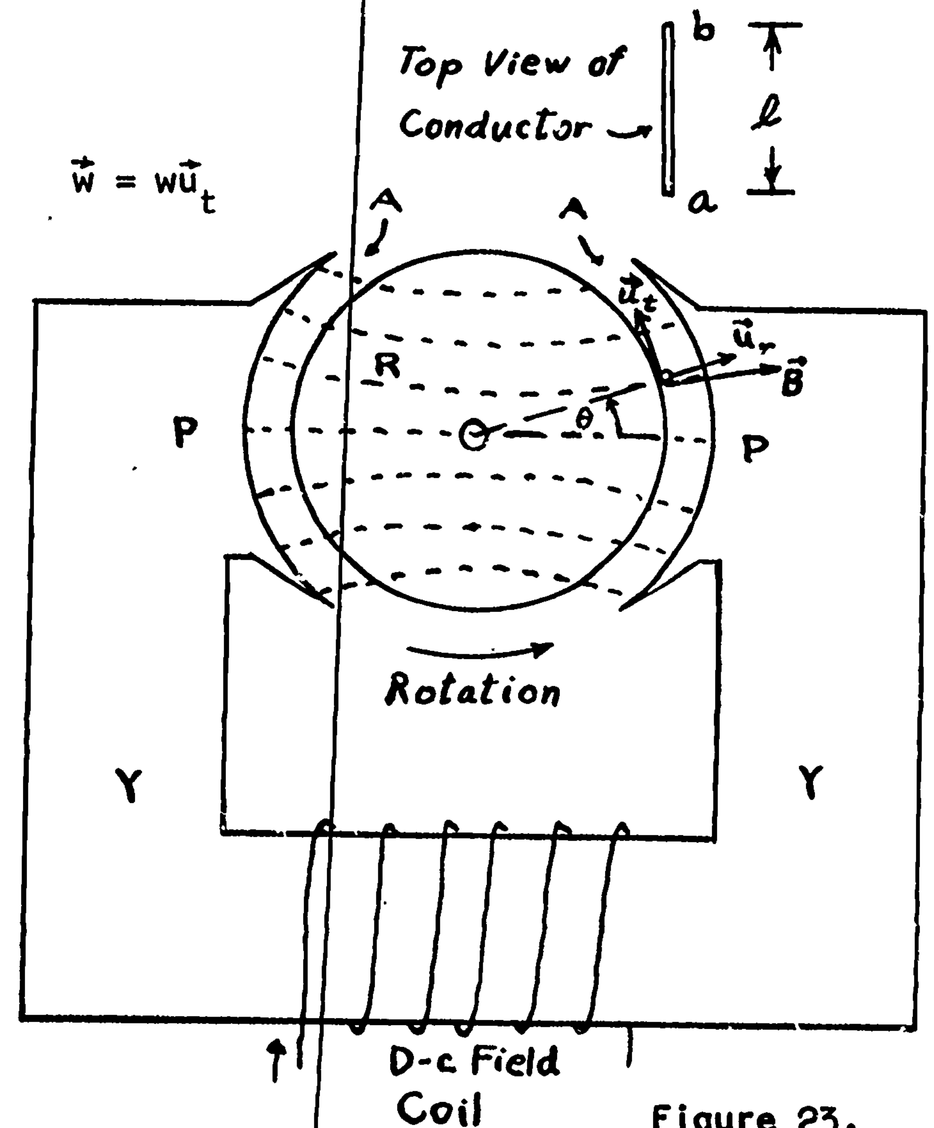


Figure 23.

The generalization developed in the previous frame is needed in order to discuss the generation of emf in an actual generator which includes a magnetic circuit consisting of a yoke (Y), pole pieces (P), air gap (A), and rotor (R), as shown in Fig. 23. Flux is produced by direct current in the field coil.

Lines tangent to the $\vec{B}$ vectors (also called lines of force) will be curved like the dashed lines shown in the figure.

Imagine that a conductor a-b (later to be a loop side) is attached to the rotor, which is rotating. For every value of θ in the range $-\pi \leqq \theta \leqq \pi$, B will have a specific value and direction, which is not necessarily radial.

The radial component (scalar component) is

$$B_r = \underline{\hspace{4cm}}$$

and

$$e_{ab} = \underline{\hspace{4cm}}$$

Answers:

$$B_r = \vec{B} \cdot \vec{u}_r$$

$$e_{ab} = \ell w(\vec{B} \cdot \vec{u}_r)$$

$$\text{or} \quad \ell w B_r$$

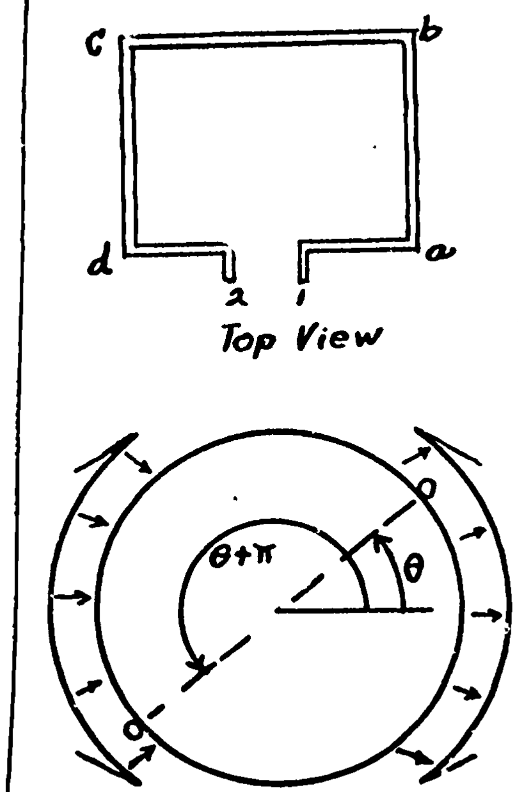


Figure 24.

Some typical $\vec{B}$ vectors are shown in Fig. 24. From this figure we see that the algebraic sign of B_r is

$$\_\_\_\_\_\_\_\_\_\_ \text{ when } 0 \leqq |\theta| < \frac{\pi}{2}$$

and

$$\_\_\_\_\_\_\_\_\_\_ \text{ when } \frac{\pi}{2} < |\theta| \leqq \pi$$

In fact, because of symmetry, if B_r has a particular value B_r' at θ, then its value at $\theta + \pi$ will be __________. Thus, if we introduce a second conductor c-d, to form a loop as before, for all positions of this loop it will be true that

$$e_{dc} = \quad e_{ab} \quad \text{(insert sign)}$$

and since

$$e_{12} = e_{ab} \quad e_{dc} \quad \text{(insert sign)}$$

it follows that

$$e_{12} = \_\_\_\_\_\_\_\_\_\_$$

<u>Answers:</u>

positive

negative

$- B_r{}'$

$e_{dc} = - e_{ab}$

$e_{12} = e_{ab} - e_{dc}$

$e_{12} = 2e_{ab}$

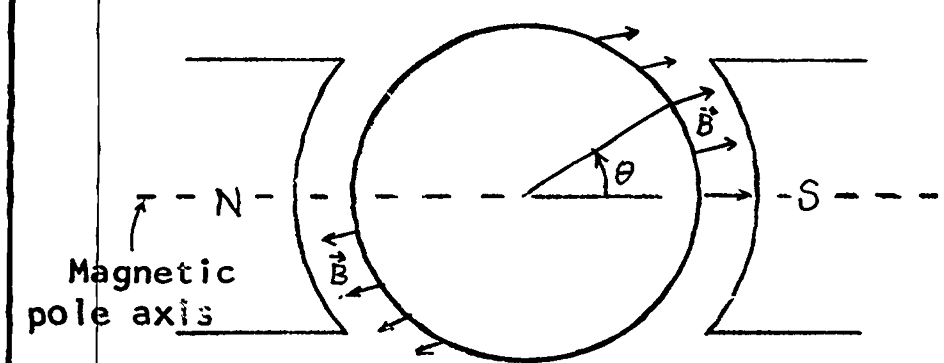


Figure 25.

Since

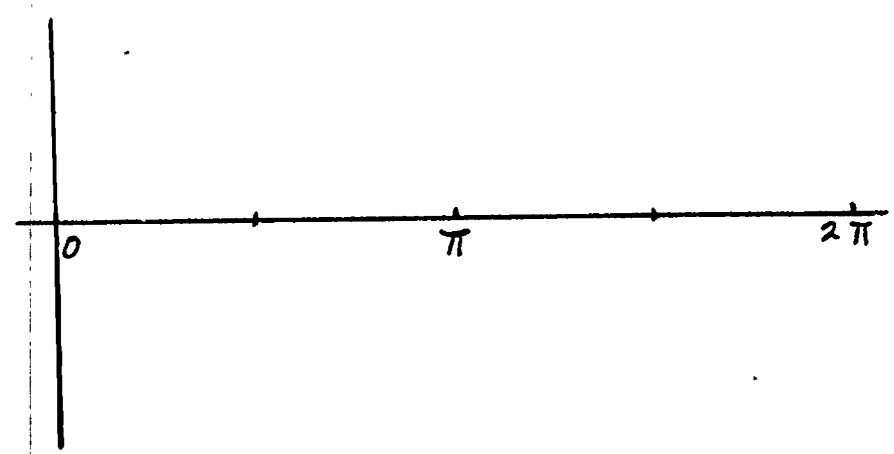
$$e_{12} = 2e_{ab} \qquad \text{and} \qquad e_{ab} = \ell w B_r$$

if the loop rotates at constant velocity, w will be a constant and therefore

$$e_{12} \text{ is } \underline{\hspace{4cm}} \text{ to } B_r$$

In fact, the standard laboratory procedure for determining B_r is to obtain the graph of e_{12} on an oscilloscope.

Indicate on the axis below how you think B_r in Fig. 25 will vary with θ.



<u>Answers:</u>

proportional

Your graph should look something like Fig. 26a, having the following features:

(1) Symmetrical with respect to π.

(2) Symmetrical, but with change in sign, with respect to $\pi/2$ and $3\pi/2$ (this is called <u>odd</u>, or <u>skew</u> symmetry).

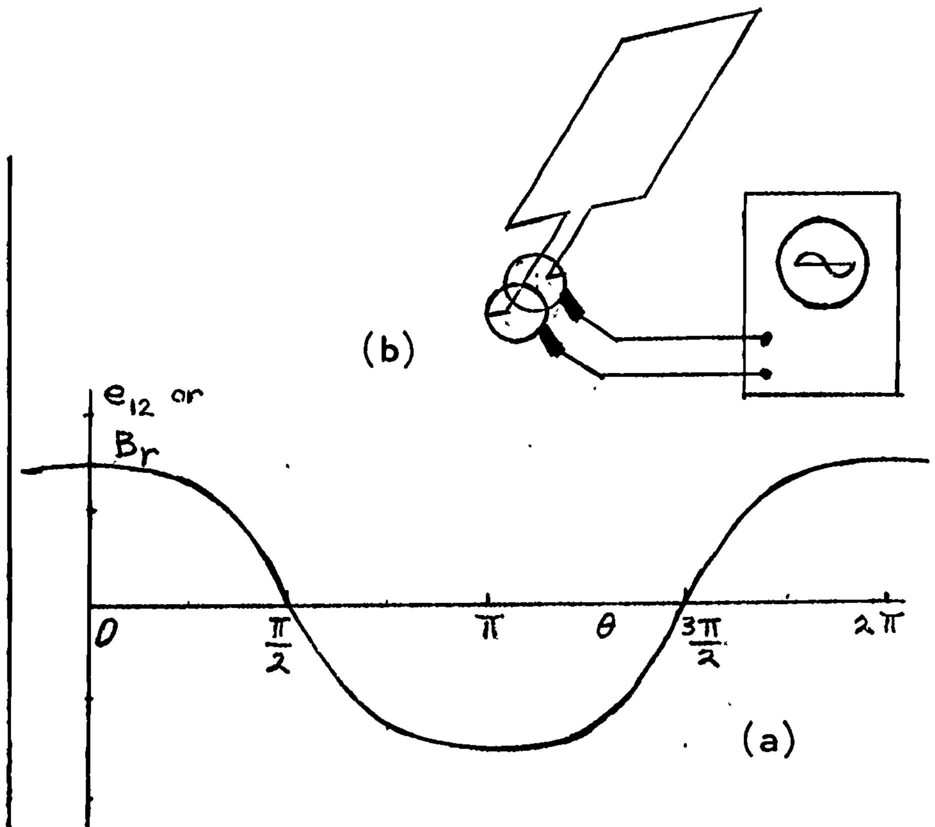


Figure 26.

Figure 26a is a picture of the voltage wave seen on an oscilloscope connected to the loop via slip rings, as shown in Fig. 26b. Such a machine is ____ ________________ ______________ generator. However, the wave shape is not ____________________ , in the manner expected from your experience with a-c circuit analysis.

Sketch such an expected wave in Fig. 26a, giving it the same peak values as the wave shown. In order to modify the shape of B_r so that it more nearly approximates your curve, the magnitude of B_r at points off the magnetic pole axis ($\theta = 0$ and π) should be ____________________.

70

Answers:

an alternating current

sinusoidal

reduced

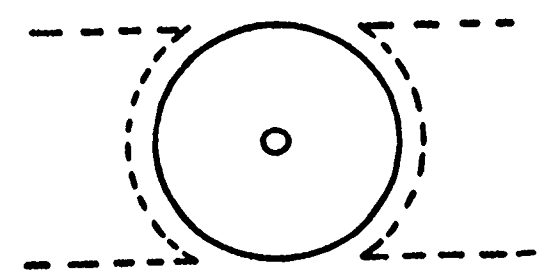
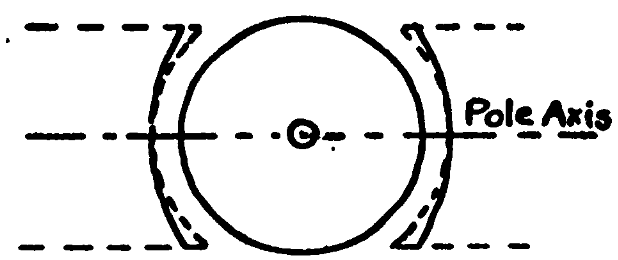


Figure 27.

Figure 27 will help to show that a more nearly sinusoidal wave can be obtained by modification of the air gap. The dotted lines represent outlines of the pole faces which give the original wave shape (not the one <u>you</u> drew) in Fig. 26a. In Fig. 27 draw in <u>modified</u> <u>pole</u> <u>face</u> <u>contours</u> that would tend to give an improved (i.e. more sinusoidal) wave shape.

This technique is actually used in the design of certain types of a-c machines.

Answer:



Increase the airgap progressively with increasing distance from the pole axis, to reduce B_r off the axis.

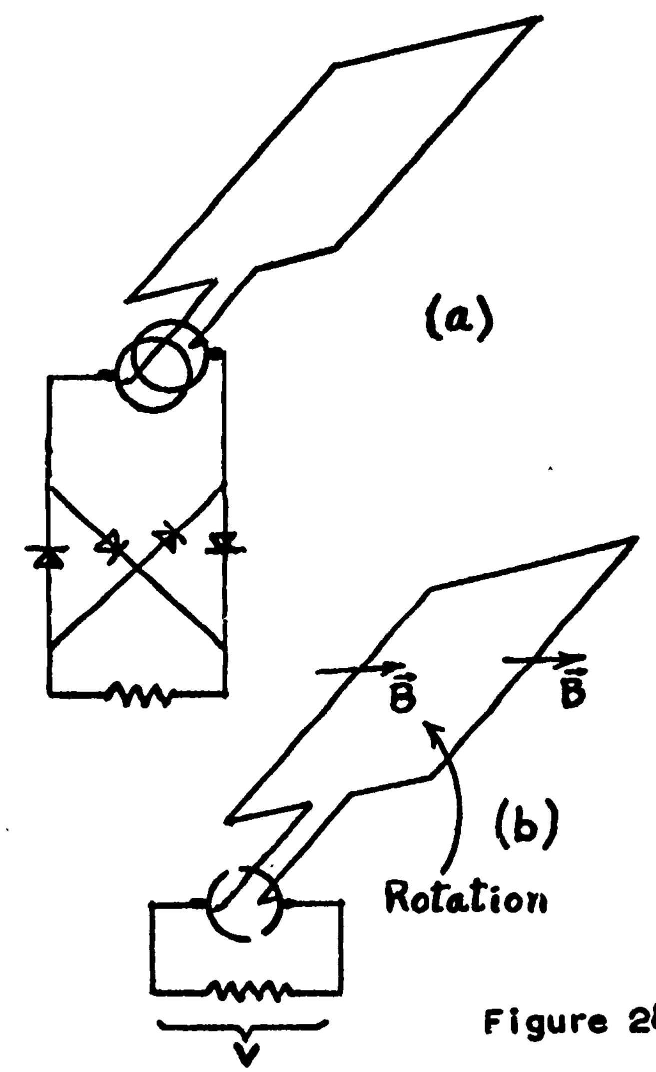


Figure 28.

Although what we have just described is not a practical a-c generator, it embodies the essential features. Before going further into practical details, we should also consider how <u>direct</u> <u>current</u> can be obtained from such an elementary machine.

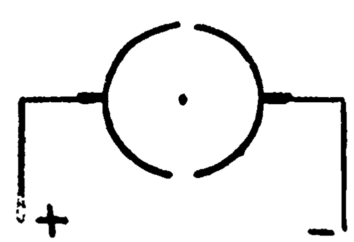
The obvious answer is to incorporate a rectifier. One way to do this would be in an <u>external</u> <u>circuit</u> using ________________, as shown in Fig. 28a. This circuit performs the function of ________________ the connections when the wave passes through zero.

The same function can be performed by replacing the slip rings by a two-segment <u>commutator</u> as shown in Fig. 28b. Mark + and - signs on the terminals, at the v symbol, indicating the polarity to be expected for the condition shown.

<u>Answers:</u>

diodes

reversing (or interchanging)



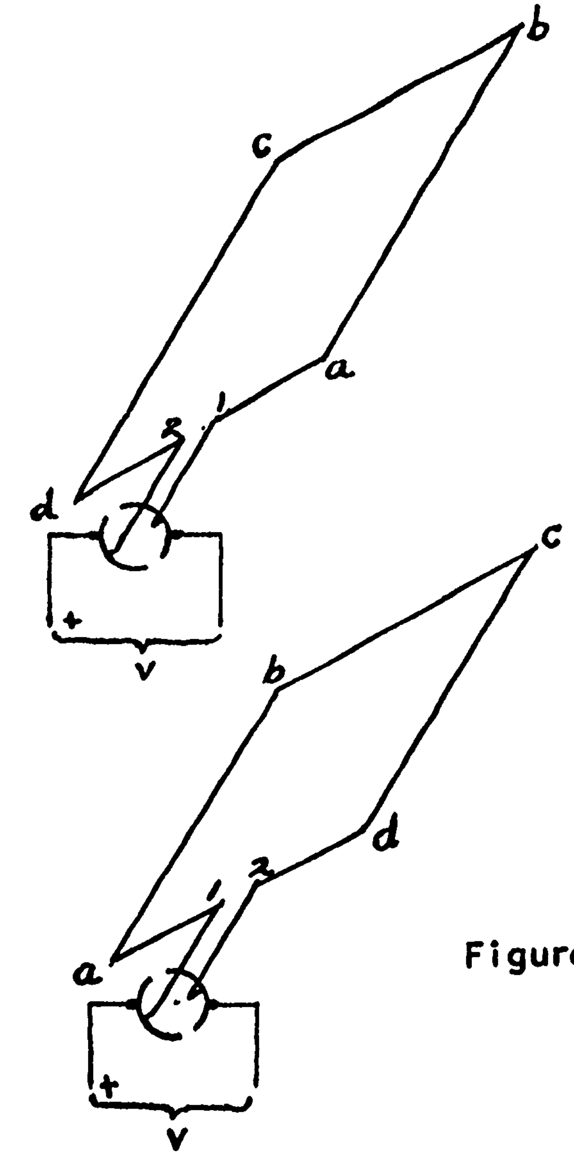


Figure 29.

It should be rather obvious that the commutator performs the function claimed for it. Nevertheless, let us examine its function critically, with reference to Fig. 29.

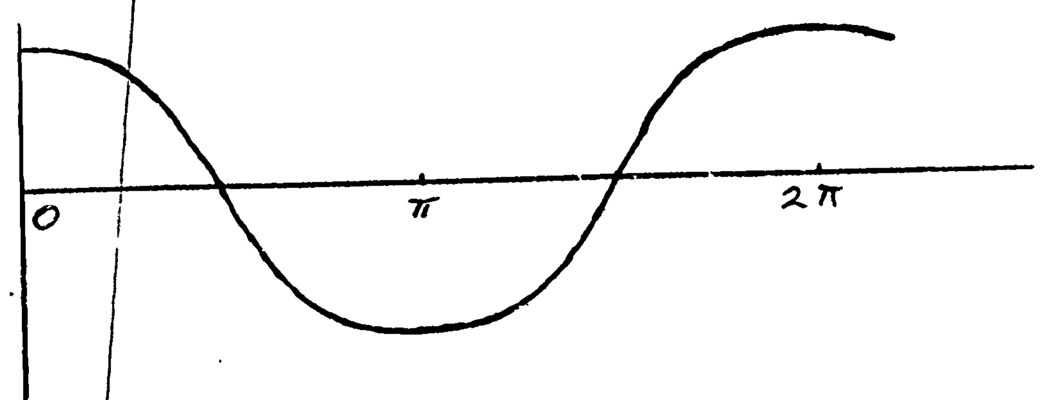
The graph of e_{12} is shown below. When the loop has the position shown in Fig. 29a,

$v =$ ________ (in terms of e_{12})

and when the loop is in position (b),

$v =$ ________ (in terms of e_{12})

As a result of these observations, you should be able to draw the wave of v. Do it on the graph below.

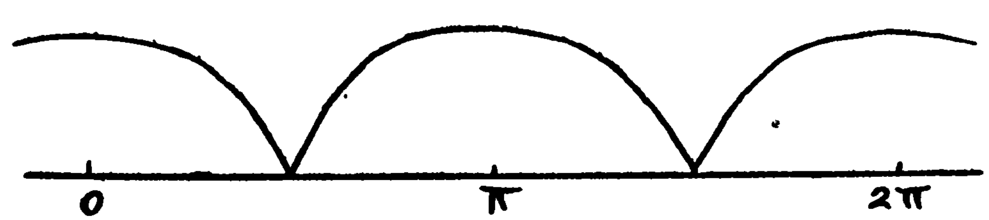


76

Answers:

$v = e_{12}$ for (a)

$v = -e_{12}$ for (b)

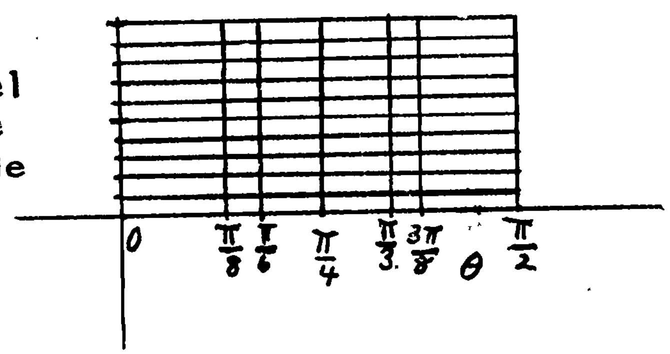


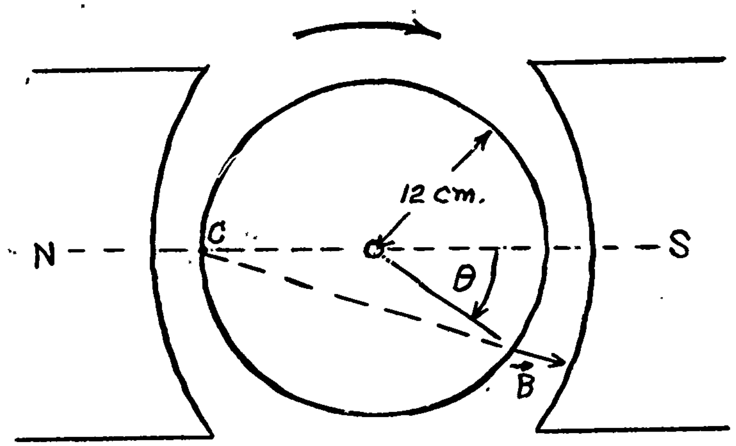
The result is a wave that is not constant, but it is always above the axis. That is, although v varies with time, it is never negative. It is the same wave shape that would be produced by the rectifier in Fig. 28a.

Our next task will be to consider some of the practical techniques of designing machines that do not have the shortcomings of the single loop machine, of which this pulsating output is one.

Table for Calculations:

Label the Scale





Length of Rotor 14 cm.

Rotational speed 1200 rpm.

Figure 30.

Review Problem

Figure 30 represents the rotor, airgap, and pole pieces of a machine in which the airgap is shaped to make the generated wave shape nearly sinusoidal. The following assumptions may be made:

(1) For $0 \leqq |\theta| \leqq \frac{\pi}{2}$, the magnitude of $\vec{B}$ in the airgap is approximated by $|\vec{B}| = 0.9(1 - \frac{3}{8}\theta^2)$ webers/sq. meter.

(2) The direction of $\vec{B}$ is along a line drawn from point C (at the intersection of the circumference of the rotor and the pole axis) to the pole face.

(3) The reference direction for e is out of the paper, and rotation is clockwise.

Calculate the emf induced in a single conductor at several positions as defined by θ, from 0 to $\frac{\pi}{2}$. Use the unlabeled table to record your results. Sketch this emf as a function of θ on the axes provided. Label your axes. Calculate and plot the values of a true sinusoidal wave at each value of θ that you used above. Compare the results from your two sets of calculations by observing the two curves.

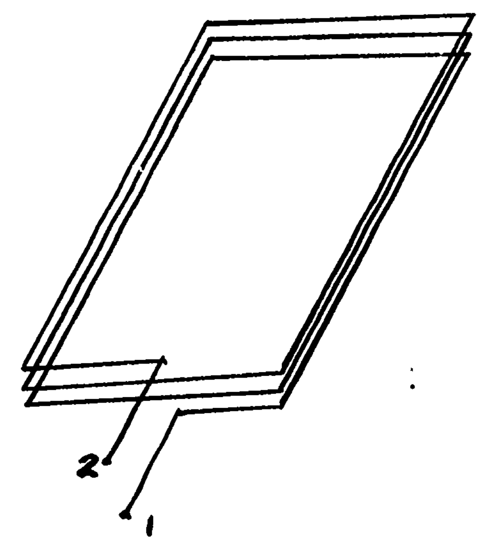


Figure 31.

Part II

The first part of this program has concentrated on the concept of induced emf, and its relationship to terminal voltage, including the basic idea of an equivalent circuit for a conductor moving in a magnetic field. The geometrical arrangements have been simple; the sliding bar and single rotating loop, including brief descriptions of the use of an iron magnetic circuit to obtain a nearly radial concentrated magnetic field in an airgap in which a loop can rotate.

Although these simple arrangements can be used to convert small amounts of energy from electrical to mechanical form, they cannot do so efficiently in the large amounts required in practice. In Part II we shall deal with some of the technological problems that have been solved in making electromechanical energy conversion a practical reality.

ARMATURE WINDINGS

For most practical applications generators operate at voltages ranging into the hundreds and thousands, although some small generators operate at lower voltages. From previous examples it can be seen that the voltage obtainable from a generator containing only a single loop will usually be much smaller than the voltage required for a practical application.

The design of practical generators has depended upon a solution to the problem of how to assemble <u>many</u> conductors on a rotor and how to connect them in series so that their emf's will add.

A fairly obvious partial solution is illustrated in Fig. 31, showing a coil of three turns. Of course, there could be any reasonable number of turns (subject to the condition of there being room for them). Say there are N of them. The voltage at the terminals of the loop will then be

$$(\quad) \times (\text{emf of one conductor})$$

82

Answer:

2N × emf

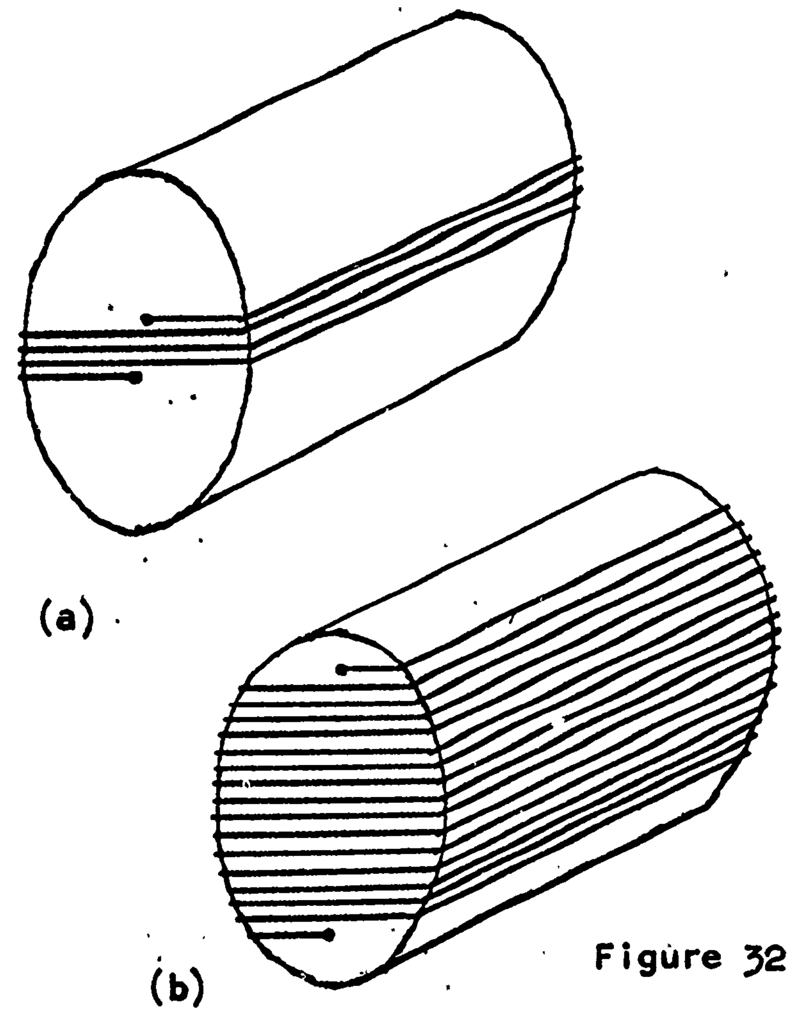


Figure 32.

However, this is only a partial solution to the problem because, a single coil such as we have just considered, with its wires all bunched together as in Fig. 32a, does not cover very much of the rotor surface. The available space on the rotor surface is not used efficiently.

In order to remedy this, one might think of winding a coil like the one shown at (b) in the figure, covering nearly all of the rotor surface.

This accomplishes the purpose in a way, but if the coil ends are connected to a two-segment commutator the resulting voltage will still have the pulsating characteristic of the single loop. As we shall see next, a slightly different method of winding will permit the use of a commutator of many segments, and a more nearly constant voltage output.

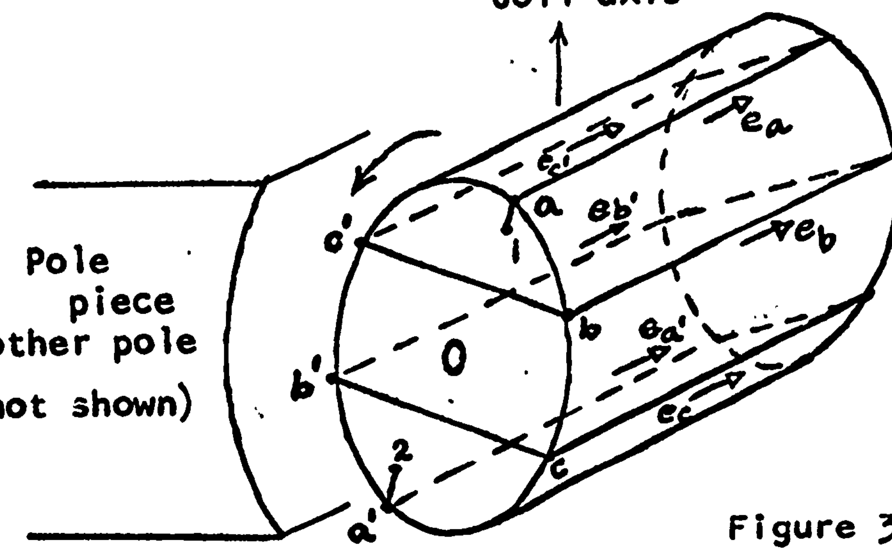
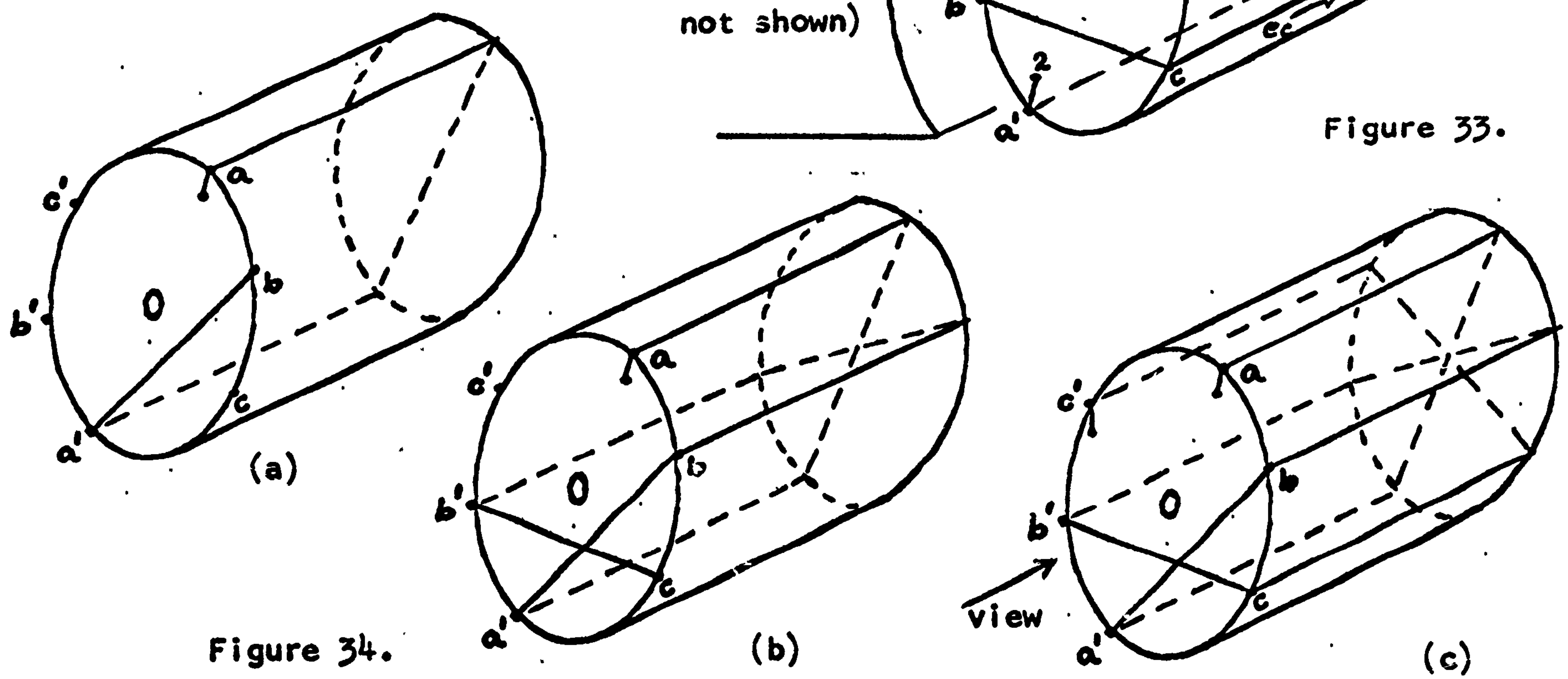


Figure 33.



(a) (b) (c)

Figure 34.

Figure 33 is a reptition of Fig. 32b, but drawn with a small enough number of turns (fewer than in practice) to permit making uncluttered drawings. The practical difficulty with this method of winding is that the resulting coil has a definite axis, as indicated by the arrow, and still permits the use of only two commutator segments. An arrangement is desired that will be symmetrical with respect to the axis of the cylindrical rotor.

The solution is obtained by spacing the sides of a winding around the circumference of the rotor, as shown in Fig. 34, and forming each turn with sides which are diametrically opposite.

If you begin at terminal (1) in Fig. 33, and add the emfs indicated by the arrows, in the sequence they are encountered, the result is

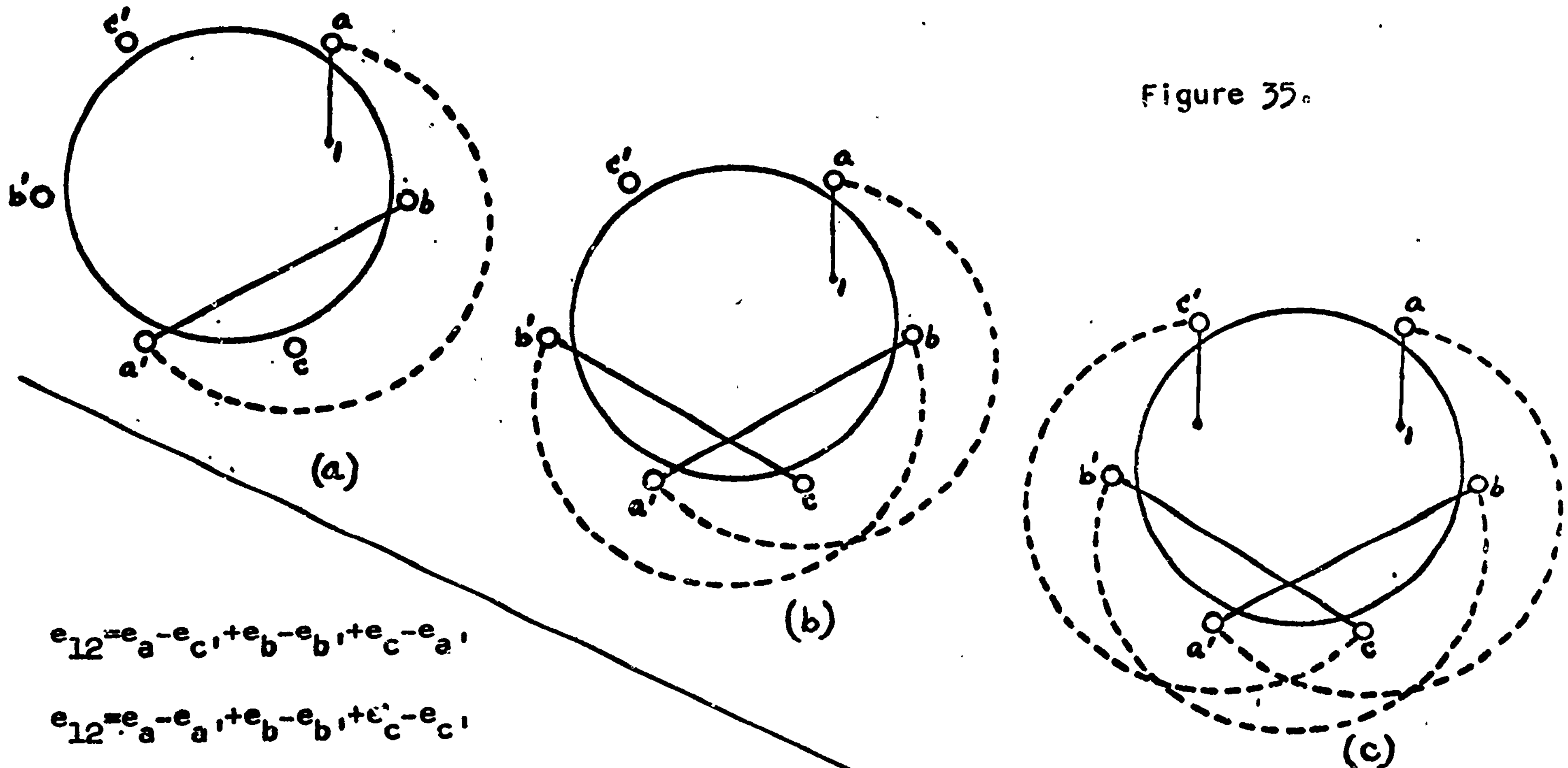
$$e_{12} = \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_$$

Assuming the same emfs also apply in Fig. 34c (not shown), for that figure we get

$$e_{12} = \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_\ \_\_\_$$

These are ____ ______.

Figure 35.



$e_{12}=e_a-e_{c'}+e_b-e_{b'}+e_c-e_{a'}$

$e_{12}=e_a-e_{a'}+e_b-e_{b'}+e_c-e_{c'}$

The same

Figure 34c still lacks the symmetry we desire, because starting and ending points (a) and (c') are different than points b, c, b' and a'. Before proceeding to investigate how to complete the winding, we shall discontinue use of the perspective view, substituting for it the symbolic picture shown in Fig. 35. In this picture, the loop sides appear as end views represented by small circles. They are viewed in the direction of the arrow marked "view" in Fig. 34. Cross connections at the back end of the rotor (shown dotted in Fig. 34) are represented by the dotted circular arcs in Fig. 35. Parts (a), (b), and (c) of Fig. 35 correspond exactly to their respective parts in Fig. 34.

With this correspondence, you should be able to recognize the method of drawing these windings as shown in Fig. 35. This portrayal will be used in subsequent diagrams.

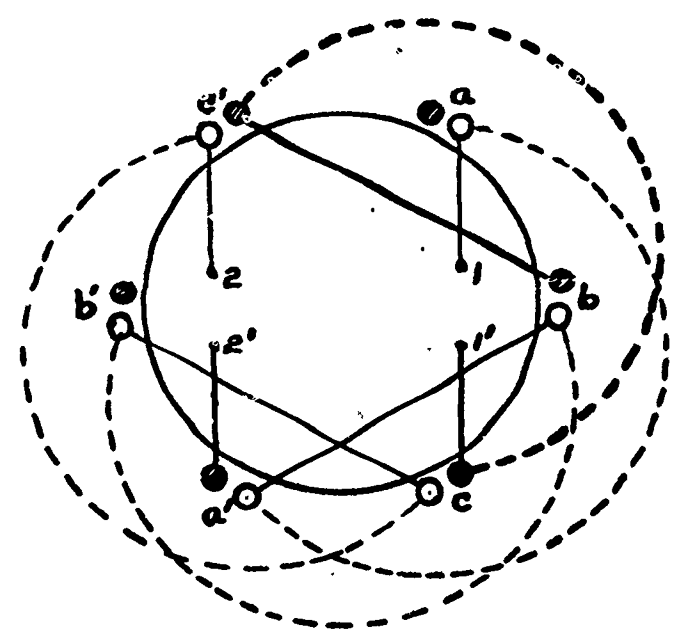


Figure 36.

Having established a method of representation, we are ready to refer to Fig. 36, in which the lightly drawn lines are the same as Fig. 35c. The symmetry we are seeking can be attained by adding <u>three more loops</u>. The sides of these loops are placed close to the ones already introduced, and are represented by the filled-in circles in Fig. 36. Starting from point 1', the <u>beginning</u> of this second winding is included in the figure. <u>You</u> draw the lines needed to complete this winding on the figure, ending up at point 2'. Do it now!

Having done this, the next step is to find $e_{1'2'}$, the emf acting between points 1' and 2'. In doing this, we can use e_a as the emf for the new (shaded) conductor which is close to the old (open circle) conductor, and similarly for the others. (This is slightly in error, because the conductors are slightly separated, but we will neglect this error.)

Recalling that the reference directions for all emf's are into the paper (see Fig. 33), taking the loop sides in sequence, you will get

$$e_{1'2'} = \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_ \quad \_\_\_\_$$

How does this compare with e_{12}? ____________________

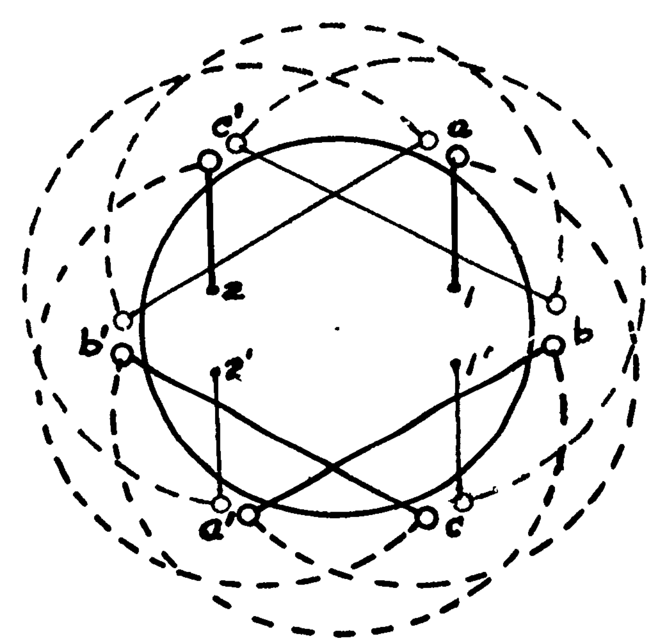


Figure 37.

The completed Fig. 36 is reproduced in Fig. 37, with the second winding shown in red.

We have made the important observation that e_{12} and $e_{1'2'}$ are identical. We are interested to know what will happen if terminals 1 and 1' are connected together, and terminals 2 and 2' are also connected together.

Which of the following do you think is true, regarding flow of current in the resulting closed circuit?

(1) Current will flow, of amount e_{12}/resistance of 1 winding.

(2) Current will flow, of amount $2e_{12}$/resistance of 1 winding.

(3) Current will flow, of amount e_{12}/2x(resistance of 1 winding).

(4) No current will flow.

<u>Answer:</u>

Choice (4) is correct.

If you picked any others, go to p. 93.

If you picked the correct answer, go to p. 95.

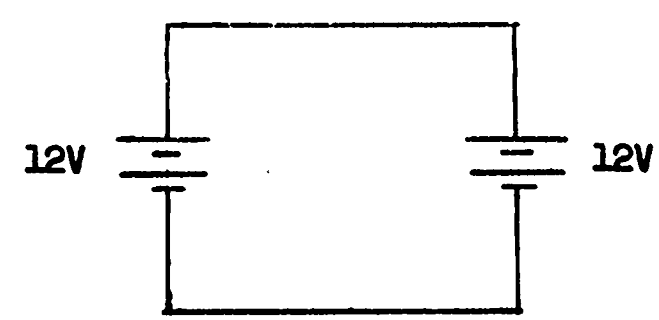


Figure 38.

You have been directed to this frame because you thought that some current would flow in Fig. 37 when connections are made between 1 and 1', and between 2 and 2'.

It is true that making these connections causes a closed loop of wire to be formed, and hence a current flow might be expected. However, the magnitude of this current will be

(__________________________) use words in this
(   ) expression

However, the total emf in the loop is

e_{12} $e_{1'2'}$ = insert sign and complete the equation.

Thus, the current will be ________.

An analogous circuit is shown in Fig. 38. As a result of the opposition of the polarities of the two batteries, the current in this circuit will be zero.

<u>Answers:</u>

$$\frac{\text{(total emf in the loop)}}{\text{(total resistance of the loop)}}$$

$e_{12} - e_{1'2'} = 0$

zero.

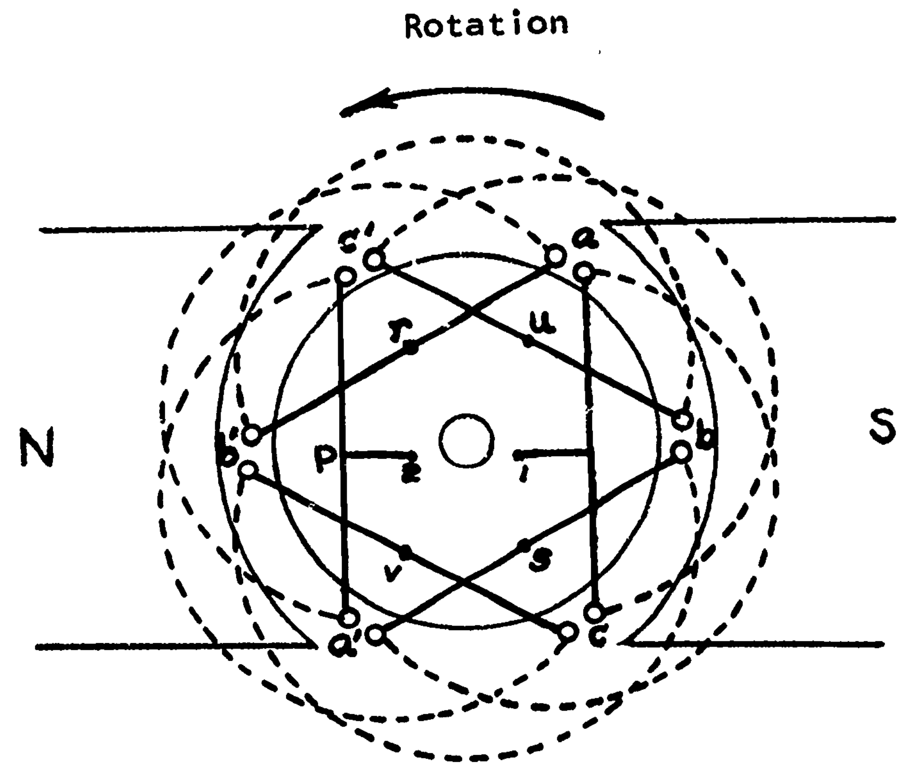


Figure 39.

The rotor, complete with the winding just described and shown in Fig. 39, is called an <u>armature</u>. Up to this point, we have two conclusions:

(1) The winding forms a closed circuit within itself, but no current flows in this circuit.

(2) An emf given by

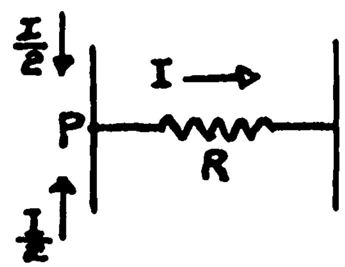
$$e_{12} = e_a - e_{a'} + e_b - e_{b'} + e_c - e_{c'} = e_a + e_b + e_c - (e_{a'} + e_{b'} + e_{c'})$$

will be found when tracing from point 1 to point 2 through either of two paths.

It follows that if a resistor R is connected between these points, a current I will flow, since this resistor will complete a circuit <u>external</u> to the armature in which an emf is acting. Draw in this resistor on Fig. 39 and along side it draw an arrow to represent the instantaneous direction of I, for the $\vec{B}$, direction of rotation, and position of the armature shown.

Also draw two current arrows, one above and one below point p, showing current direction in each of the parallel paths, and label them in terms of I.

Answer:



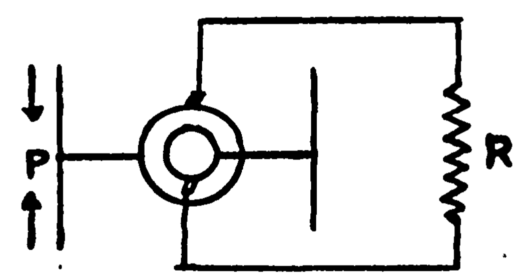


Figure 40.

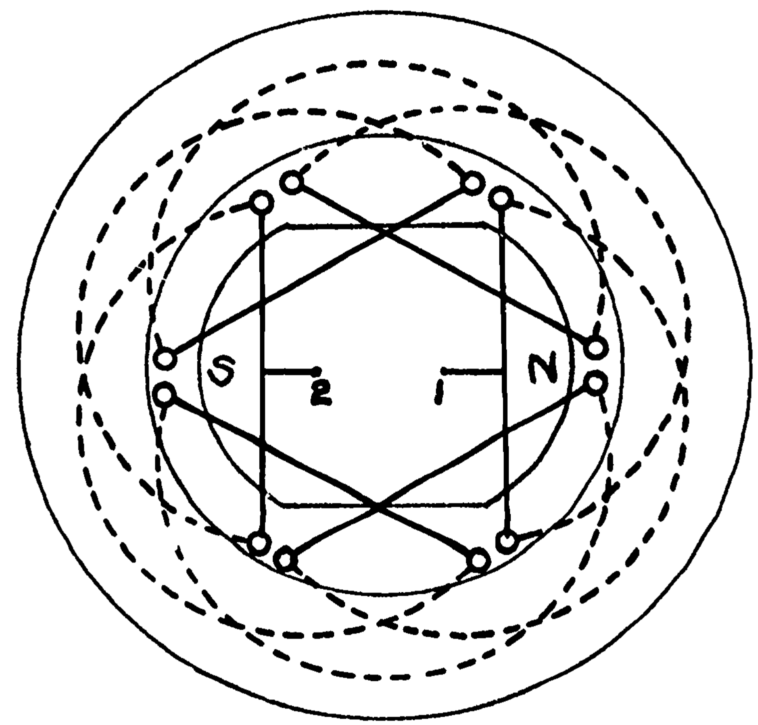


Figure 41.

To avoid the practical difficulty of having the load attached to the rotor, slip rings can be used as shown in Fig. 40, to give a practical form of a-c generator. However, sliding contacts are troublesome, particularly when large currents must be carried by the slip rings. Accordingly, the arrangement shown in Fig. 41 represents standard design practice. The windings are identical with Fig. 39, but they are on a stationary iron ring, while a <u>two-pole</u> <u>magnet</u> <u>rotates</u> <u>inside</u>.

This could be a permanent magnet, but since adjustment of its strength is usually necessary, it is customary to make it an electromagnet, with a d-c winding with current supplied through slip rings. Since slip rings are still needed, why do you suppose they are less troublesome than in the arrangement of Fig. 40?

____

____

What should be the direction of rotation of the magnet in order to duplicate the situation portrayed in Fig. 39? ______________________
(cw or ccw)

Answers:

The d-c current required for the magnet will be very much smaller than the a-c load current. Thus, in Fig. 41 the slip rings can be much smaller.

Clockwise.

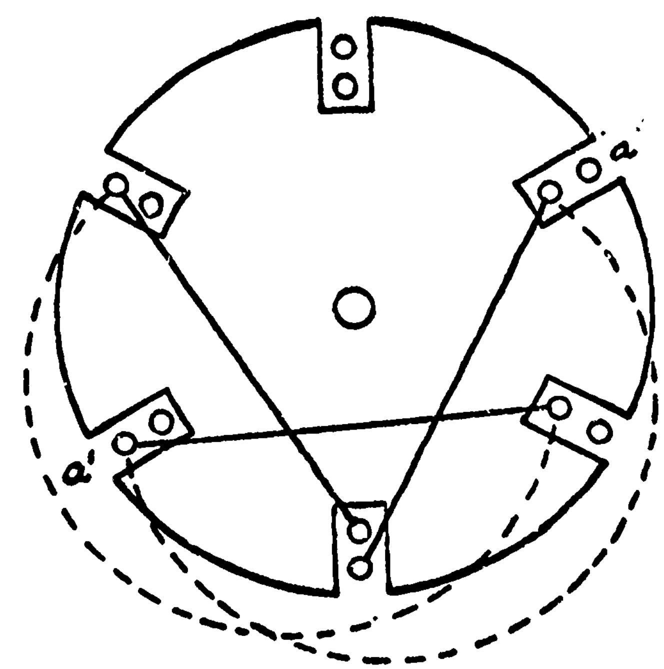


Figure 42.

Having shown that it makes no essential difference whether the coils rotate in a stationary magnetic field, or are stationary in a rotating magnetic field, we shall continue to consider only the case of rotating coils. Two additional constructional features of armature windings will be mentioned.

(1) Conductors are placed in slots on the rotor (or stator) rather than on the surface, as we have shown them. Furthermore, surface space is conserved by placing the two adjacent loop sides above one another, as shown in the partially constructed winding in Fig. 42. It is a point of particular interest to observe that one side of a loop is on the inside of a slot, while the other side is on the outside.

(2) Up to this point it has been assumed that the lines on the wiring diagrams represent single wires. For example, in Fig. 42, it has been assumed that (a) and (a') represent the sides of a single loop of wire. This might actually be the case for a very low voltage generator. But in most cases, what we have

(Over)

previously called a loop is usually a coil. We did not consider this fact earlier, in order to keep the diagram as simple as possible. In fact, we shall not attempt to show a complete winding with coils in all positions, but Fig. 43 shows one coil consisting of three turns, in the same position as loop (a-a') of Fig. 42.

Compared with Figs. 39 and 41, the voltage obtained with N turn coils will be N times greater.

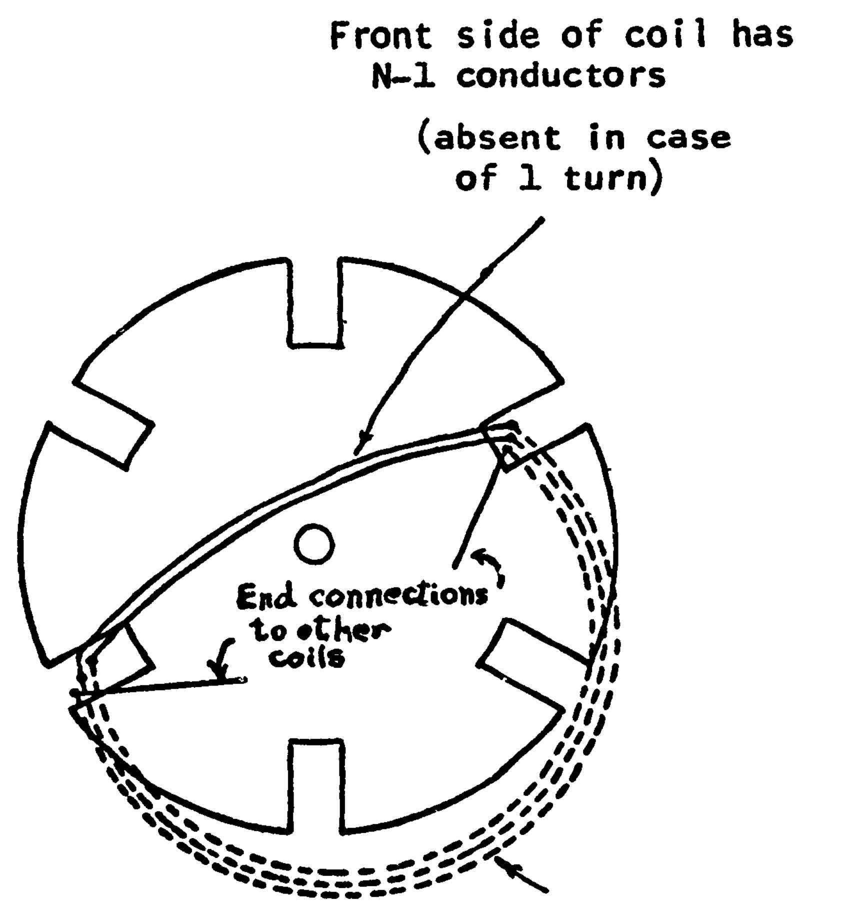


Figure 43.

This completes our discussion of typical arrangements of conductors. The treatment is sufficient for the purpose at hand, although many details have been omitted. Additional information can be found in numerous books on electric machinery. In particular, it should be noted that we have been dealing with only six loops (or coils), which is a relatively small number. It is not unusual for a large machine to have more than a hundred coils. Nevertheless, the principles of construction are exactly as described here.

We can summarize the main features as follows:

(1) There are two parallel paths through the armature, from terminal to terminal.

(2) The winding is symmetrical with respect to the rotor axis. For example, in Fig. 39 on page 94 the external terminals could be connected between points r and s, or between u and v. We shall make use of the symmetry proprety when we return to a consideration of d-c machines.

If wound like Fig. 39,

$$e_{12} = e_a - e_{a'} + e_b - e_{b'} + e_c - e_{c'}$$

$$= e_a + e_b + e_c - (e_{a'} + e_{b'} + e_{c'})$$

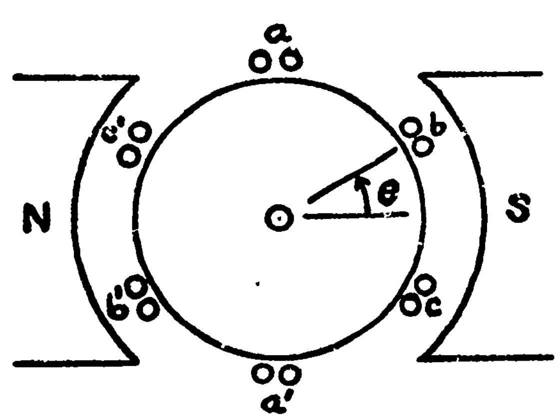


Figure 44.

Reference direction of each e is into paper, in Fig. 44.

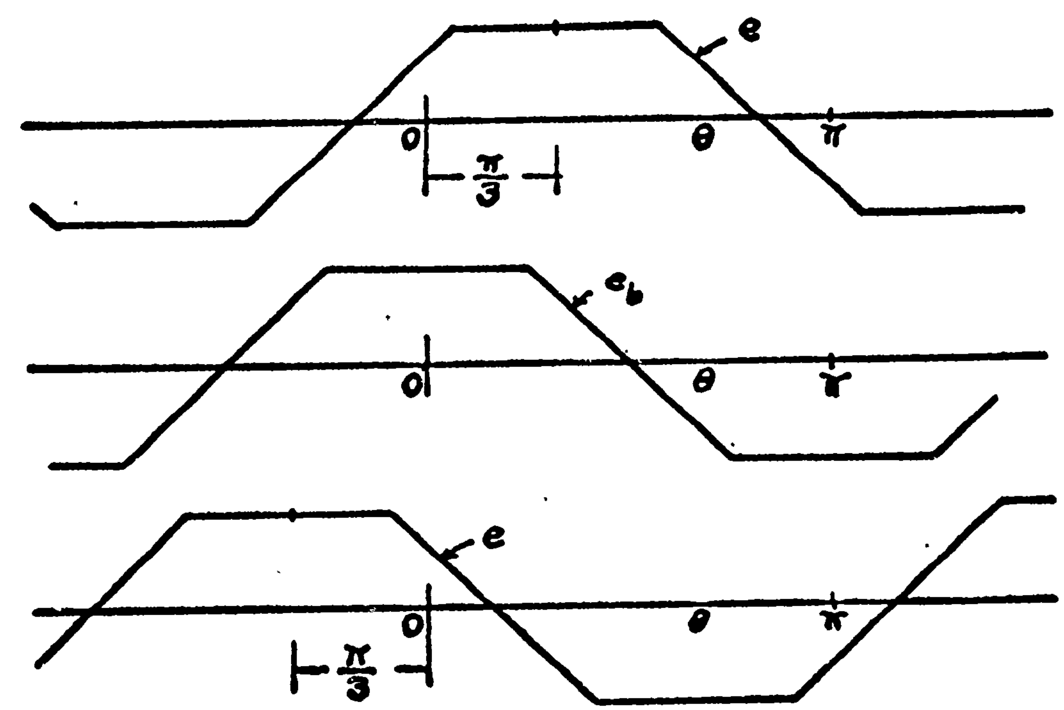


Figure 45.

Now we shall consider the analytical problem of determining how the emf of the complete winding varies as the rotor changes position. We are still dealing with Fig. 39, but its essential features are repeated in Fig. 44, with the rotor moved counter-clockwise through the angle θ, compared with the earlier figure. We shall determine how e_{12} varies with this angle.

First you should look at the graph labeled e_b in Fig. 45. This is a hypothetical plot of how e_b varies with θ, when rotation is at constant speed. It is constructed of straight lines to simplify the discussion, but in reality this would be a relatively smooth (but not quite sinusoidal) wave like Fig. 26.

Conductors (a) and (c) go through the same process of rotation as (b), but are displaced from it by $\pi/3$ radians. Thus, the curves for e_a and e_c will be similar to the curve for e_b, but will be shifted along the θ axis, as shown in Fig. 45.

Place appropriate labels (e_a and e_c) on the two unlabeled curves in Fig. 45.

Answer:

See labels at right.

Observe that the positive peak of e_c occurs when (c) is on the pole axis, which is when $\theta = \pi/3$. Likewise, e_a is at its positive peak when (a) is on the pole axis, which occurs when $\theta = -\pi/3$. This is consistent with the choice of labels shown.

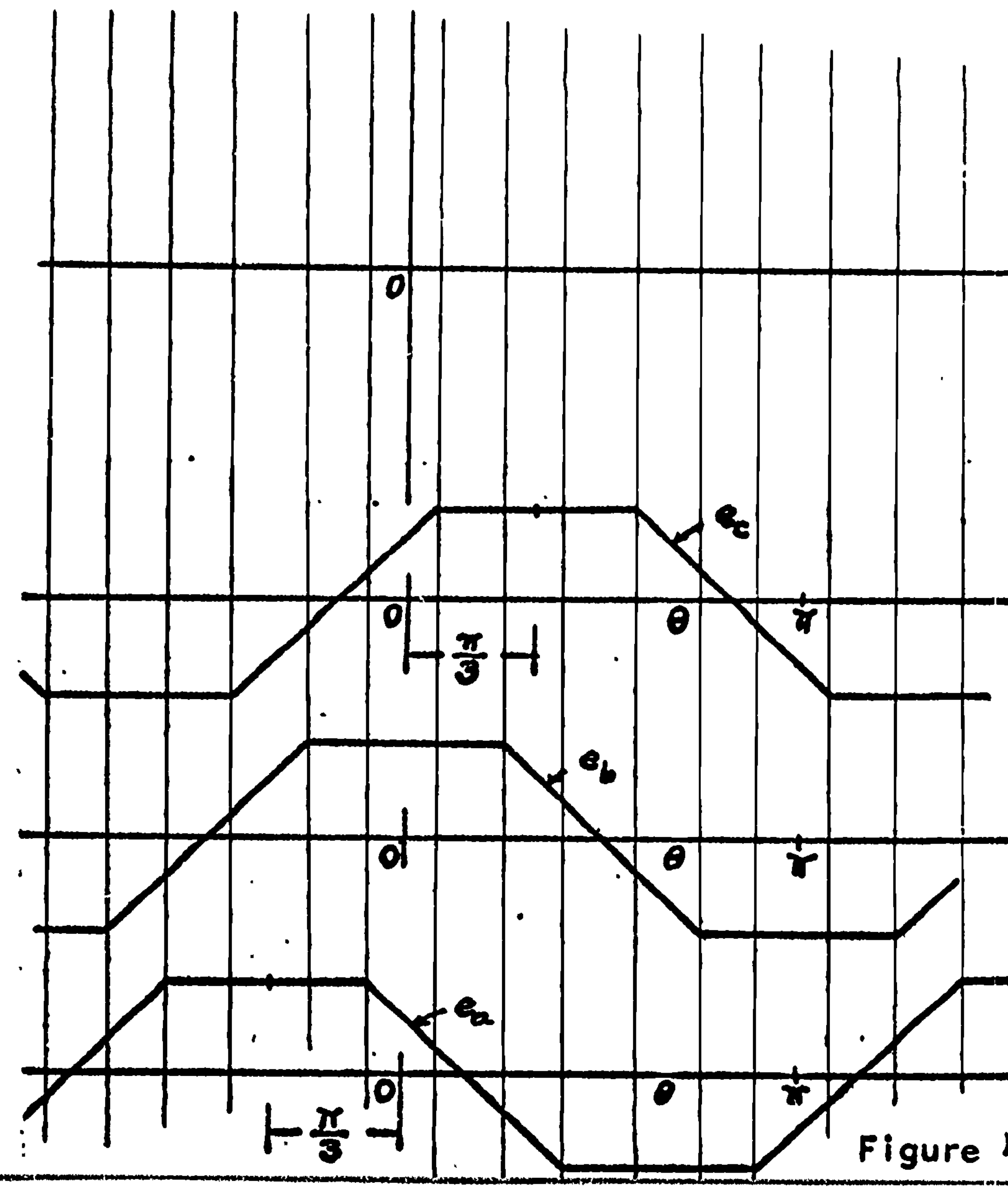


Figure 46.

Next we combine e_a, e_b, and e_c to obtain e_{12}, recalling that

$$e_{12} = e_a - e_{a'} + e_b - e_{b'} + e_c - e_{c'} = (e_a+e_b+e_c)-(e_{a'}+e_{b'}+e_{c'})$$

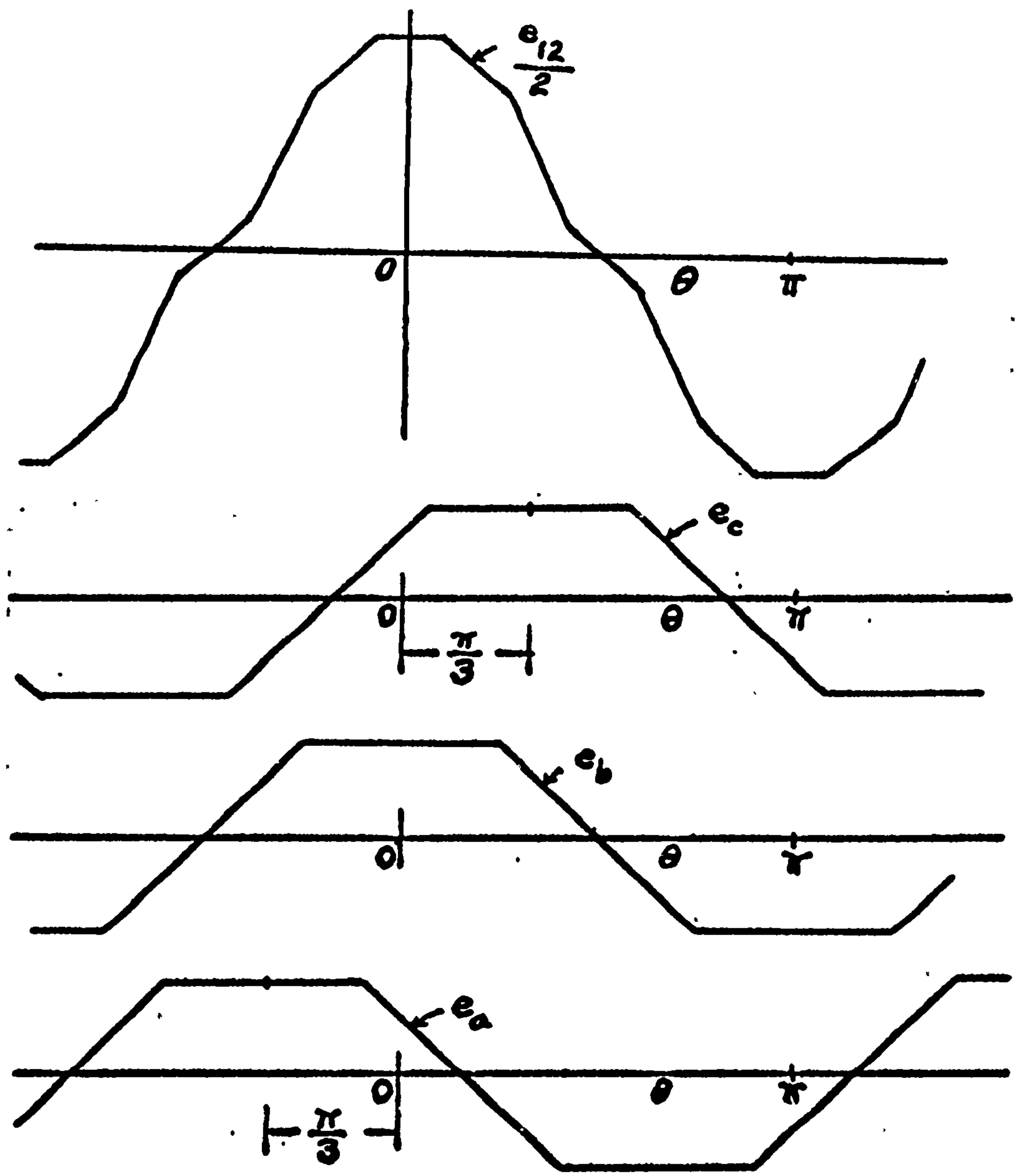
But first we note that the quantities in parentheses are negatives of each other. (Do you know why? If not, go to page 109.) Therefore,

$$e_{12} = 2(e_a+e_b+e_c)$$

showing that, except for the factor 2, it is sufficient to add only three emfs.

Figure 46 is constructed to facilitate a graphical determination of $e_a + e_b + e_c$, at the values of θ indicated by the vertical coordinate lines. Plot the sum on the set of axes shown above, and draw the graph of $e_{12}/2$.

Hint: The required addition can easily be accomplished with sufficient accuracy by marking the individual ordinates on the edge of a piece of paper. You will find it is necessary to do this only a few times, because the wave is symmetrical.



$\frac{e_{12}}{2}$
0
θ
π
e_c
$\frac{\pi}{3}$
e_b
e_a

Are you surprised by the result? The combined wave is much more nearly sinusoidal than the individual ones from which it is obtained. This is a fortunate result, which can be explained mathematically through the use of Fourier series (which we shall not do here). It is generally true that the greater the number of coils that are distributed around the periphery, the more nearly the output wave will approximate a sinusoid. By a combination of this principle, and use of an airgap design which gives a nearly sinusoidal variation of B, very accurate sinusoidal waves can be generated in practice.

Now observe that, although we have plotted a wave as a function of θ, since rotation is at constant speed, θ is proportional to time t. For example, suppose $t = 0$ corresponds to $\theta = 0$, and rotation is at 3600 rpm. What is the value of t when $\theta = \pi$? ______________

Go to p. 111.

Answer:

.00833 sec.

Now go to p. 111

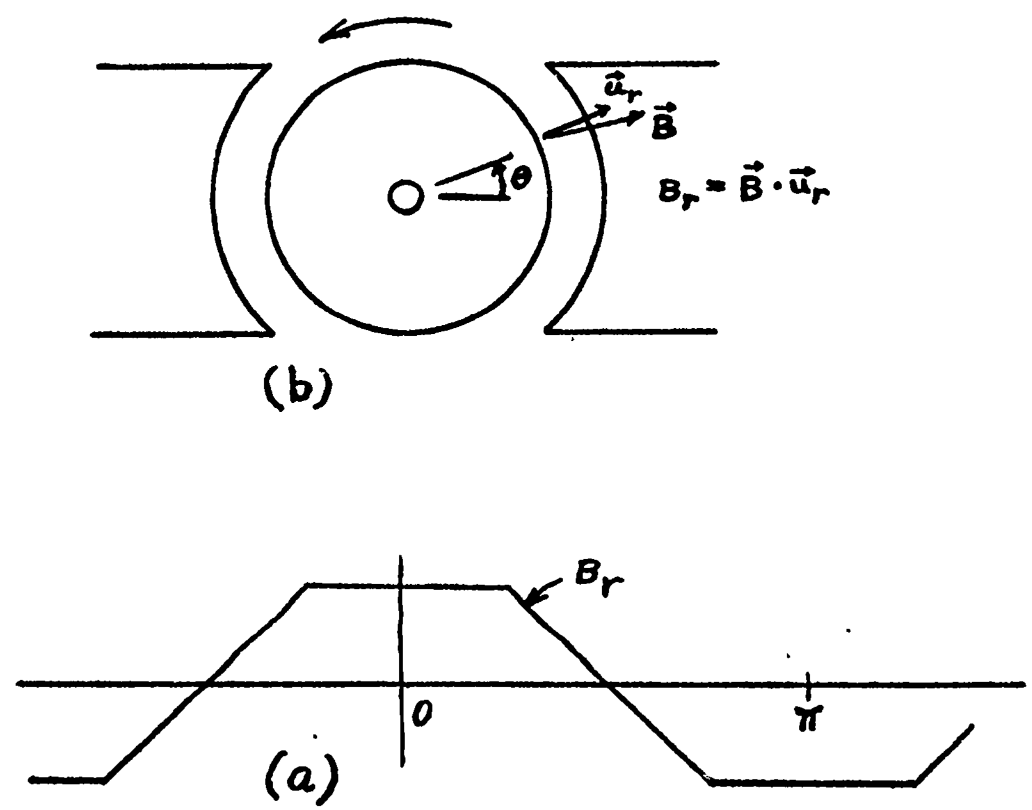


Figure 47.

You have been referred to this page for a proof that in Fig. 39 (or its derived form, Fig. 44),

$$e_a + e_b + e_c = -(e_{a'} + e_{b'} + e_{c'})$$

This is true because we have assumed that the flux density wave B_r is symmetrical, as in Fig. 47a. Thus, to use conductors (a) and (a') as an example, since they are π radians apart, their emfs e_a and $e_{a'}$ (which are proportional to B_r) will <u>always</u> be the negative of each other. Thus,

$$e_a = - e_{a'} \quad , \quad e_b = - e_{b'} \quad , \quad e_c = - e_{c'}$$

and adding these gives the desired result.

Go to p. 105.

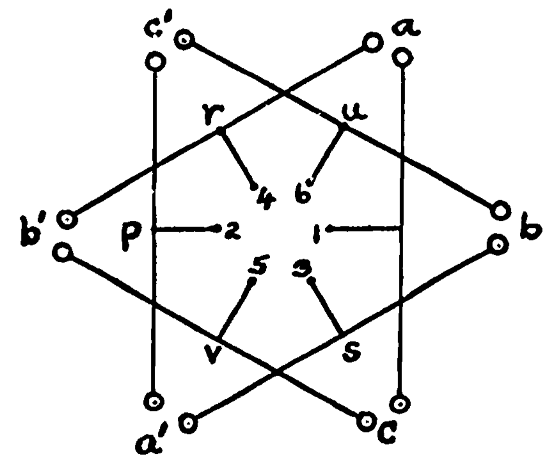


Figure 48.

Having found that e_{12} is nearly sinusoidal, let us next consider what difference it would make if the slip rings were connected to r and s, and u and v, of Fig. 48. This figure is a reproduction of the end connections in Fig. 39. For example, let us determine the wave of e_{34}.

The answer might be obvious to you, as a result of symmetry of the wingings. If not, it can be obtained analytically, by writing

$$e_{34} = \_\_\_\_\ \_\_\_\_\ \_\_\_\_\ \_\_\_\_\ \_\_\_\_\ \_\_\_\_$$

$$= (\qquad\qquad) - (\qquad\qquad)$$

$$= 2(\qquad\qquad)$$

112

Answers:

$$e_{34} = e_b - e_{b'} + e_c - e_{c'} + e_{a'} - e_a$$

$$= (e_b + e_c - e_a) - (e_{b'} + e_{c'} - e_{a'})$$

$$= 2(e_b + e_c - e_a)$$

You might also have

$$2(e_b + e_c + e_{a'})$$

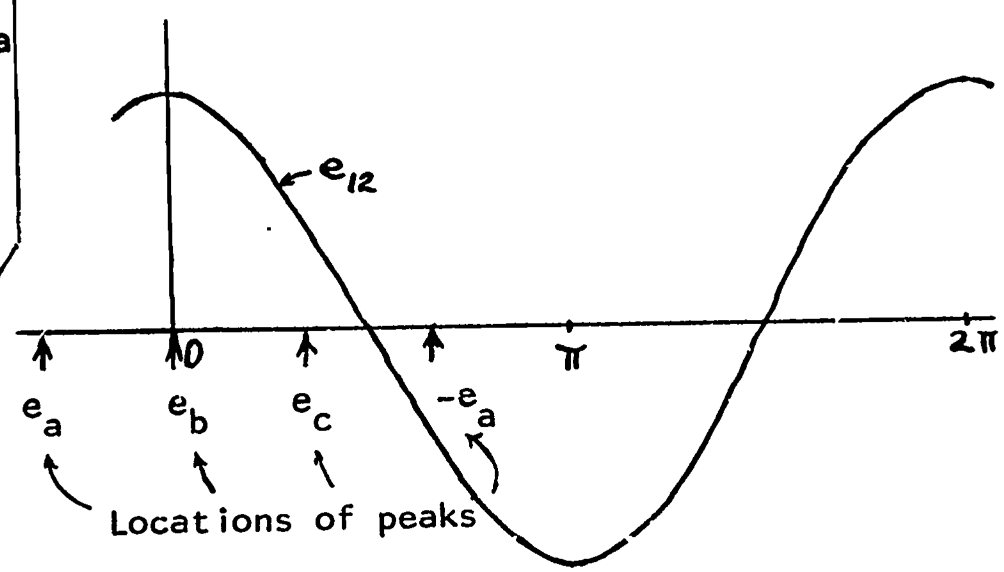
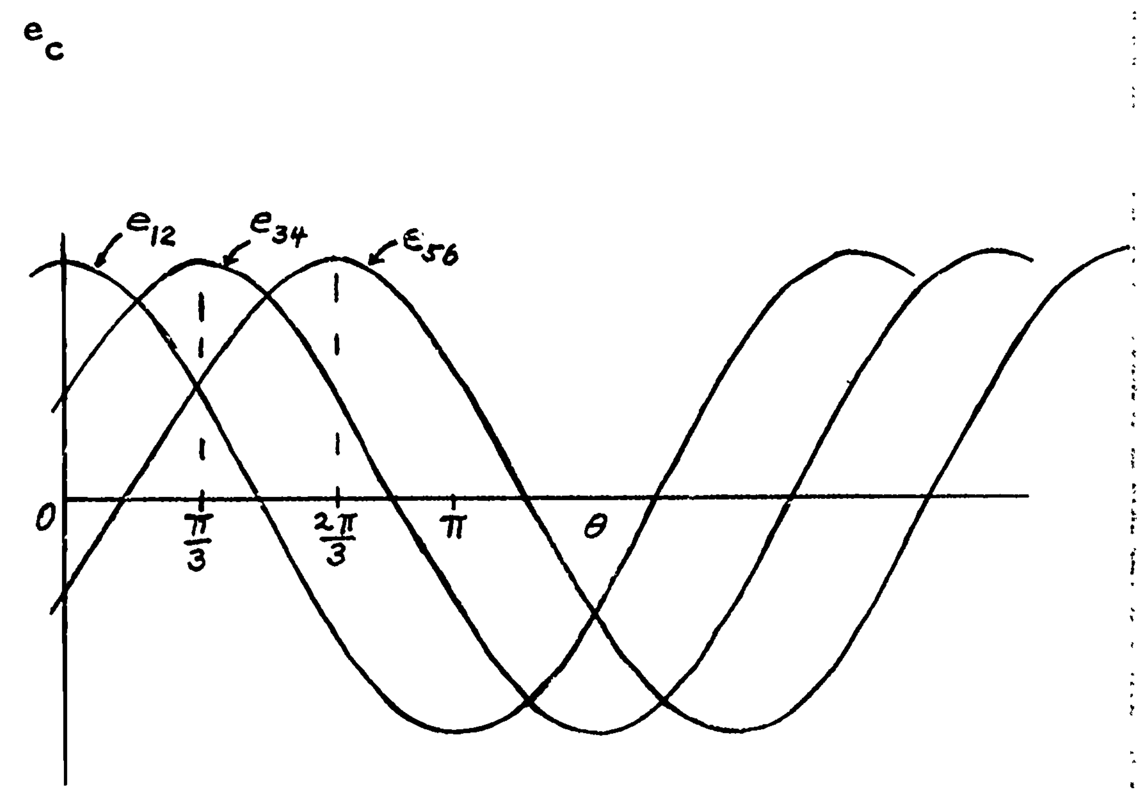


Figure 49.

To continue this discussion, let the wave labeled e_{12} in Fig. 49 be the sinusoidal approximation for e_{12} as you drew it in Fig. 46. Also, observe that the location of the positive peak of e_{12} in Fig. 46 occurs at the midpoint of the positive peak of e_b. The plot of e_b can be identified as the one of the three (e_a, e_b, e_c) whose positive peak is between the other two. This principle can be used to locate the positive peak of e_{34}, relating it to the waves of e_b, e_c, and $-e_a$. Thus, it is found that the positive peak value of e_{34} occurs at the center of the positive peak of wave ______.

Sketch e_{34} on Fig. 49, and also (from a similar reasoning process) determine where e_{56} should be, and sketch it.

Answer:



Thus, it may be said that the waves of e_{12}, e_{34}, and e_{56} are similar, but that between successive waves there is a ________ ________ of ________ radians. Furthermore, e_{12} ________ (leads or lags) e_{34} by this angle.

We have been plotting waves as a function of angle of the rotor, though on p. 107 it was mentioned that θ is proportional to ________ ________. Therefore, these waves could also be viewed as functions of time.

If rotation is at 3000 rpm, what is the frequency of e_{12}, and what is the time interval between positive peak values of e_{12} and e_{34}?

frequency = ________

time interval = ________

Answers:

phase difference

of $\pi/3$ radians

e_{12} leads e_{34}

frequency = 50 cps (3000/60)

time interval = $\frac{.02}{6}$ = .0033 sec.

since $\pi/3$ radians is 1/6 of the period, which is 2π on the θ scale or 1/(frequency) on the time scale.

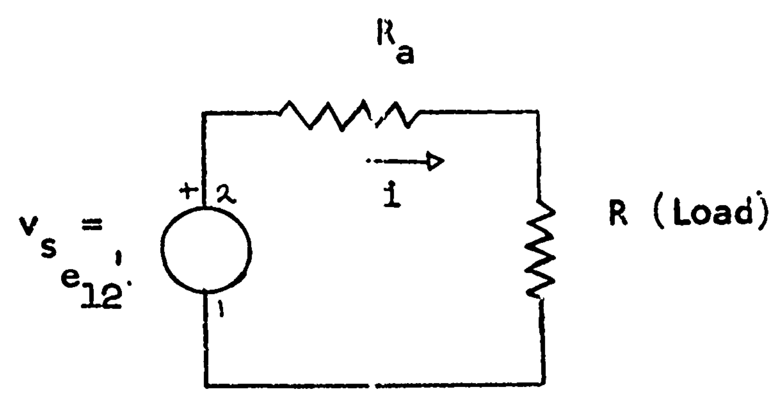


Figure 50.

$v = e_{12}' - i R_a$

R_a = armature resistance

As a last topic on alternating current generators, let us consider its equivalent circuit. We had practice on finding equivalent circuits, beginning on p. 23, for the case of a sliding bar.

Equivalent circuits are found by considering what happens when an external circuit is created by connecting a load, so that a current will flow. Figure 50 is the same as the circuit considered on p. 24, but with a change in notation. The symbol e_{12}' is the <u>actual</u> emf when armature current is flowing, and is not necessarily the same as e_{12}, the emf on open circuit. This difference between e_{12} and e_{12}' is the point of our present attention. The flux difference is due to the fact that when current flows in the armature it disturbs the airgap flux. The phenomenon whereby the flux is affected by armature current is called <u>armature reaction</u>.

A detailed study of armature reaction, including the effect of magnetic saturation, is necessary in order to analyze the behavior of an a-c generator to any degree of accuracy. However, its <u>approximate</u> effect can be obtained readily by using the

go to p. 118.

concept of self inductance.

As we have defined e_{12}' and e_{12}, it is evident that the difference between them, thought of as a single emf $(e_{12}' - e_{12})$ can be viewed as the emf induced by the time rate of change of the difference between the actual flux and the open circuit flux. But this flux difference is due to the load (armature) current. If the magnetic material were linear (straight line B-H curve) this flux difference would be proportional to armature current. This is the same as the situation encountered in the definition of self inductance of a coil, and so we can define an inductance L_a associated with the armature circuit, giving

$$e_{12}' - e_{12} = - L_a \frac{di}{dt}$$

This inductance accounts for the effect of armature reaction, although only approximately, because the B-H relationship is not linear in an actual machine.

This is the reason for the earlier statement that this is an approximate analysis.

In fact, in practice the approximation can be quite bad, leading to errors of 20% or more. However, the use of L_a, and the equivalent circuit presently to be derived from it, will predict correctly some general characteristics of a-c generators under steady state conditions. For example, it will predict correctly the <u>general</u> manner in which terminal voltage will vary with changing power factor, although the numerical values will be inaccurate.

From p. 116 $v = e_{12}' - i\,R_a$

From p. 118. $e_{12}' - e_{12} = -L_a \frac{di}{dt}$

where i is the armature current, and R_a is the armature resistance.

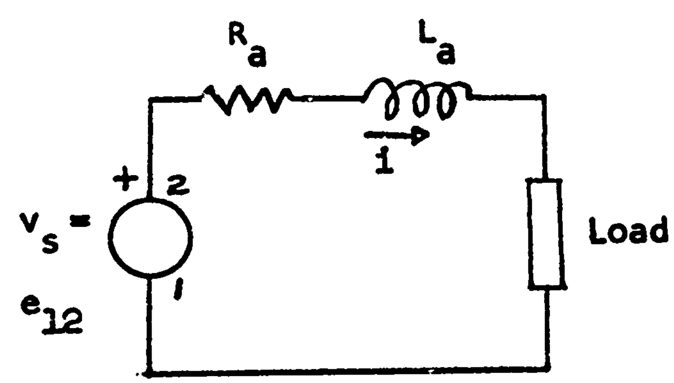
We now have the two equations shown on page 120. They can be combined to give the terminal voltage v in terms of e_{12} and i, as follows:

$$v = \underline{\hspace{5cm}}$$

This resulting equation determines the form of the equivalent circuit. Use a "box" to represent the load (since in the a-c case the load might not be a pure resistance as portrayed in Fig. 8b), and draw and label the equivalent circuit in the space below.

<u>Answer:</u>

$$v = e_{12} - i\,R_a - L_a \frac{di}{dt}$$



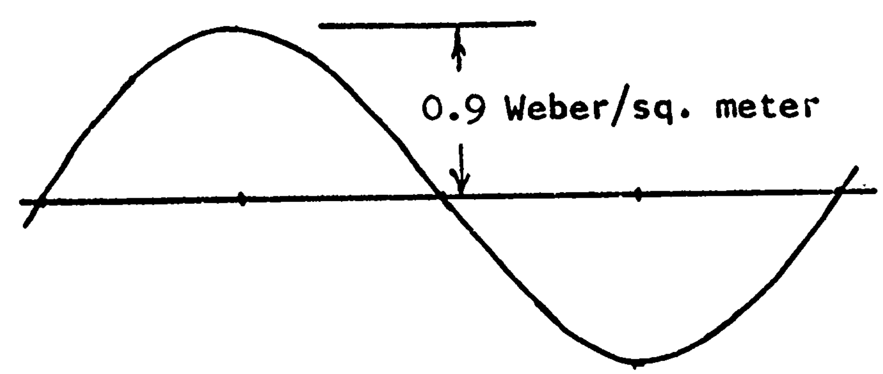


Figure 51.

Review Problem

Consider an a-c generator of the type under discussion, in which there are 12 coils on the armature, uniformly spaced as in Figs. 39 and 41, with the coil sides at a mean radius of 20 cm. Assume a sinusoidal distribution of B_r around the periphery (an idealization) as shown in Fig. 51. The length of the armature (along the axis) is 30 cm. Each coil has 4 turns of wire. The frequency is 60 cps.

(1) What is the rms value of the open circuit terminal voltage? You will need the following intermediate quantities:

Peripheral velocity = meters/sec.

Rms emf per conductor =

Rms voltage per coil =

Phase angle between coil emfs =

(2) If the wire can safely carry 25 amperes, how much current can be delivered by the machine?

Hint: Note that you can use phasors graphically, or complex numbers, to add the coil emfs.

Check you own answers. Do they seem reasonable? This problem was designed to give answers that should look reasonable to you.

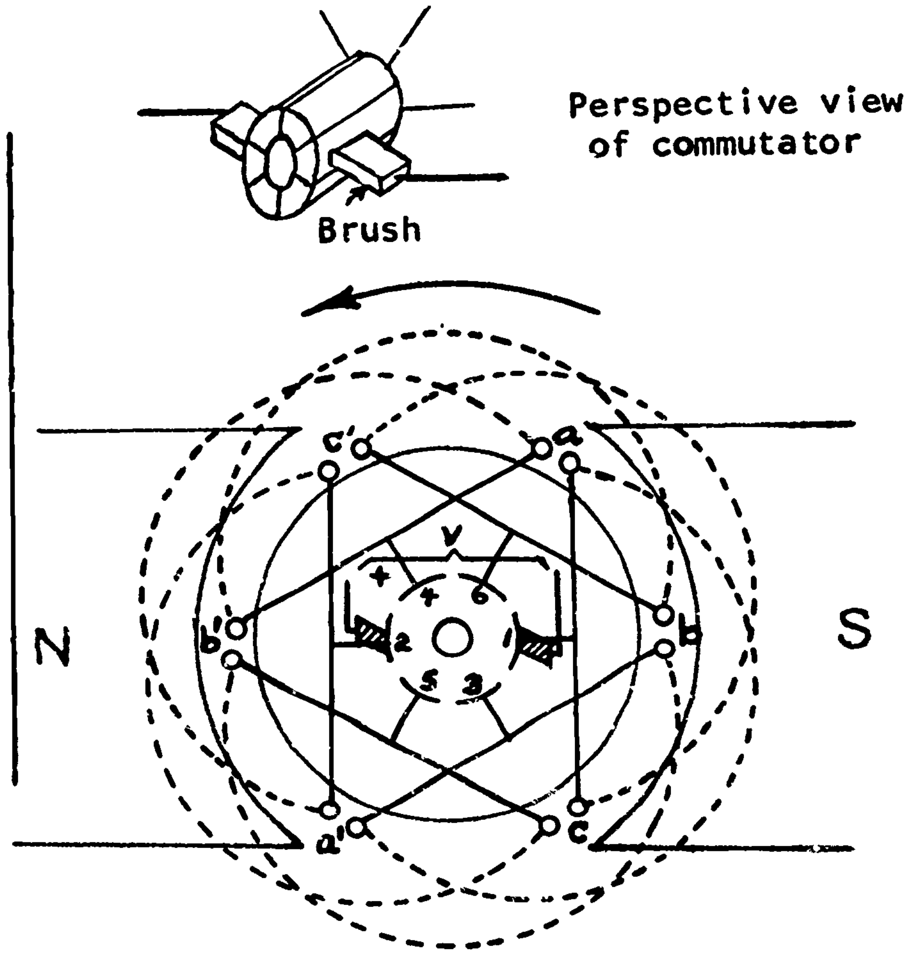


Figure 52.

D-C Generators

Beginning back on p. 73, there was a brief discussion of the operation of a 2 segment commutator for converting the a-c emf wave into one that was unidirectional, but not a constant (not pure d-c). We shall now return to the six coil winding described in Fig. 39 and repeated with the slight modification shown in Fig. 52.

A commutator for this machine consists of six segments connected to the winding by wires (represented by radial lines in the figure). The two diagonal structures touching the commutator represent "brushes" which are stationary but brush against and make contact with the commutator segments (see the perspective view). The symbol v represents the terminal voltage, with reference polarity as indicated. Let us begin by considering open circuit conditions.

For the position shown, $v = e_{12}$. In the following parentheses insert appropriate symbols (in terms of emf) which express v for successive positions, each representing an advance of 1/6 revolution from the position shown (but not including it):

(   ) , (   ) , (   ) , (   ), (   ) , (   )

Answers:

e_{34}

e_{56}

e_{21}

e_{43}

e_{65}

e_{12}

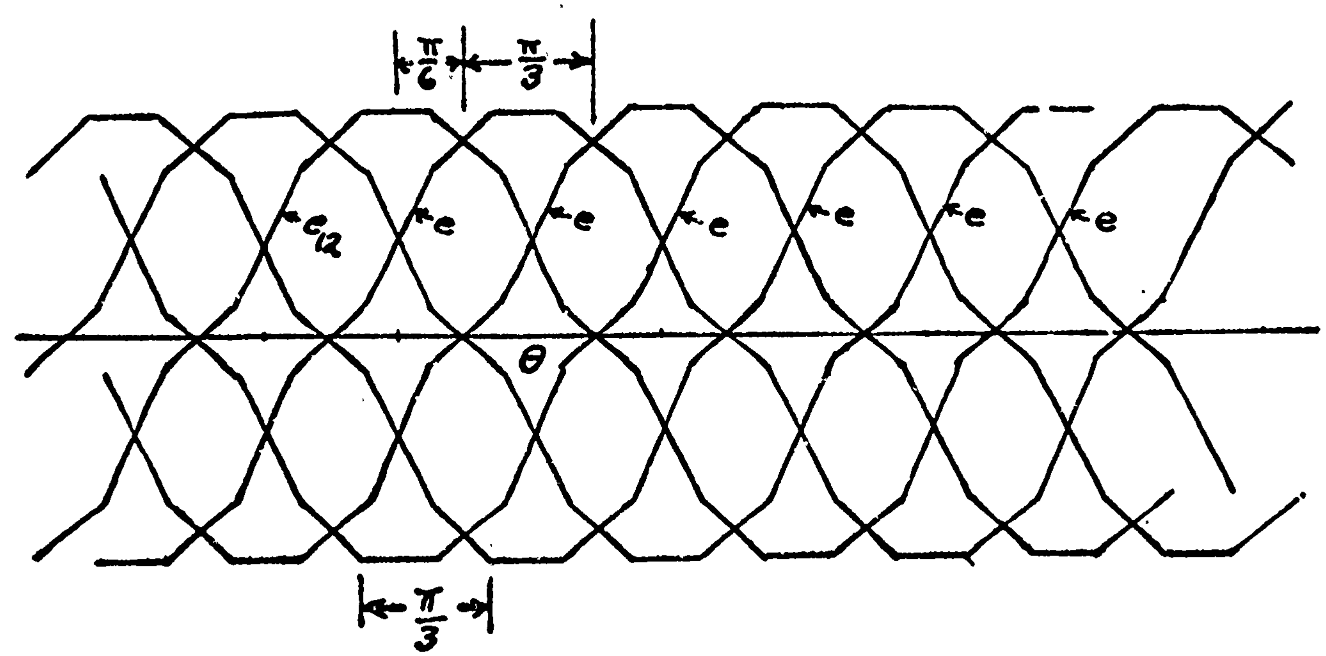


Figure 53.

3

The emf wave for e_{12} was derived on p. 105, with the result given on p. 106. The fact that we now have a commutator makes no change in e_{12}. Also, on p. 115 we reached a conclusion that there would be a phase difference of ________ radians between e_{12} and e_{34}, and between e_{34} and e_{56}.

A set of 6 waves like e_{12}, with the above phase difference between successive waves, is shown in Fig. 53.

(1) Place the proper subscripts on each of the unlabeled e symbols in Fig. 53.

(2) Using the results arrived at on p. 125, <u>trace out</u> on Fig. 53 the wave of v (the open circuit voltage defined in Fig. 51).

128

Answer:

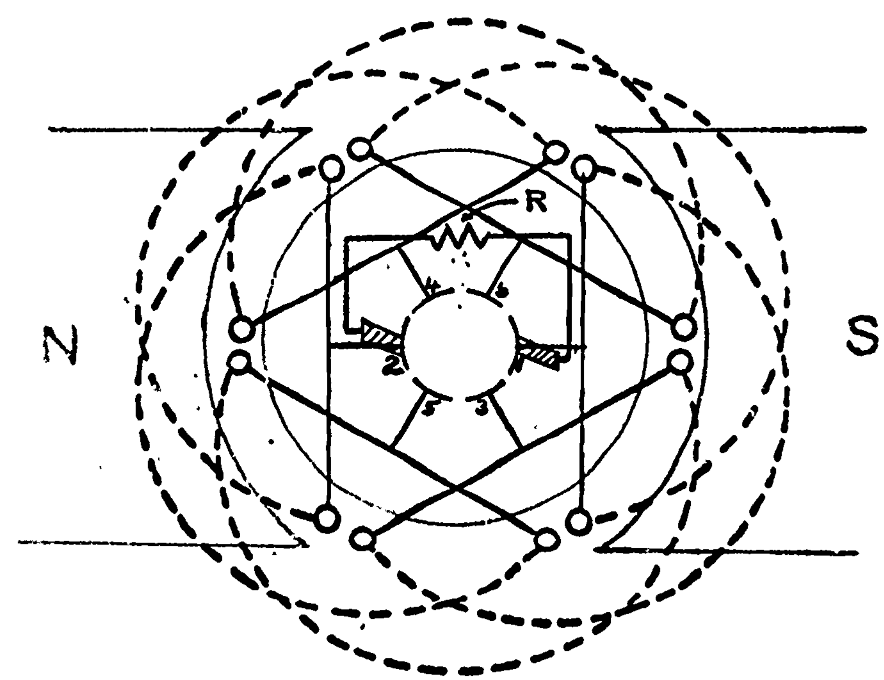


Figure 52 (repeat) with R added.

It is apparent that with six commutator segments the terminal voltage is much more nearly constant than with only two segments. The remaining fluctuation in the wave is called <u>commutator ripple</u>. The greater the number of coils, and hence commutator segments, the smaller the ripple. A d-c machine with a sufficient number of commutator segments exhibits a very nearly constant emf, equal to the peak value of any one of the e waves. We shall use E to designate this peak value.

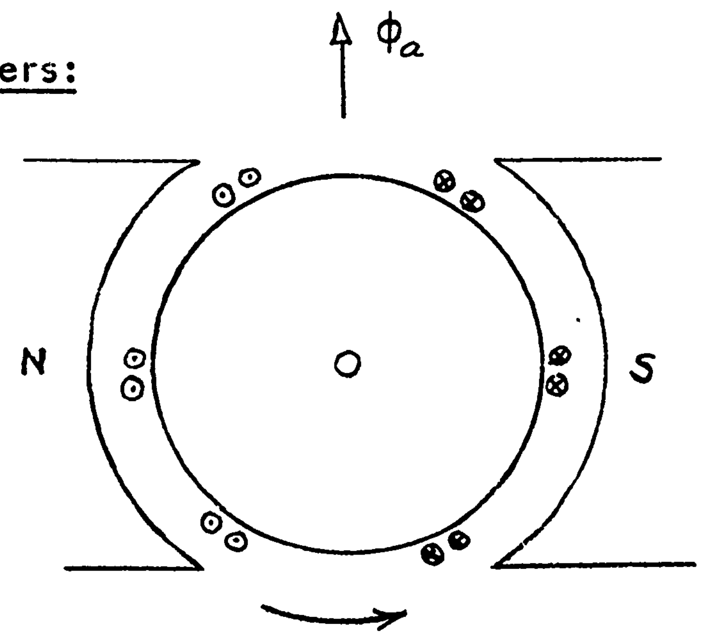
Next, let us consider what happens when a load resistor is connected to the brushes, permitting an armature current to flow. Referring to Fig. 52, place marks (· for current coming out of the paper, and x for current going into the paper) in each small circle representing a conductor.

As a result of the above, it is evident that the armature conductors act like a coil, with ampere turns tending to produce a flux in a certain direction. Show that direction by an arrow labeled ϕ_a.

After 1/6 of a revolution, the direction of ϕ_a will be ________________, and therefore we can say that ϕ_a is ________________.

130

Answers:



unchanged

constant

actually, ϕ_a will vary slightly within the $\pi/6$ sector (why?) and so a better answer would be nearly constant.

The tendency of armature current in a d-c generator to produce a "cross" field (that is, a field perpendicular to the main flux) is called armature reaction. It is different from armature reaction in an a-c machine because the armature ampere turns are nearly constant. However, armature reaction does have the effect of distorting the airgap flux, and therefore changing the shape of the wave of emf induced in each conductor. Due to nonlinear effects associated with saturation of the magnetic circuit, the distortion of the emf wave includes a reduction of its average (average over 1/2 cycle) value. (Why didn't we say average over the whole cycle?)______________________________

Armature reaction affects the equivalent circuit of a d-c generator, but in such a complicated way that the effect is most readily determined experimentally.

Answer:

The average over a whole cycle is always zero for a symmetrical wave.

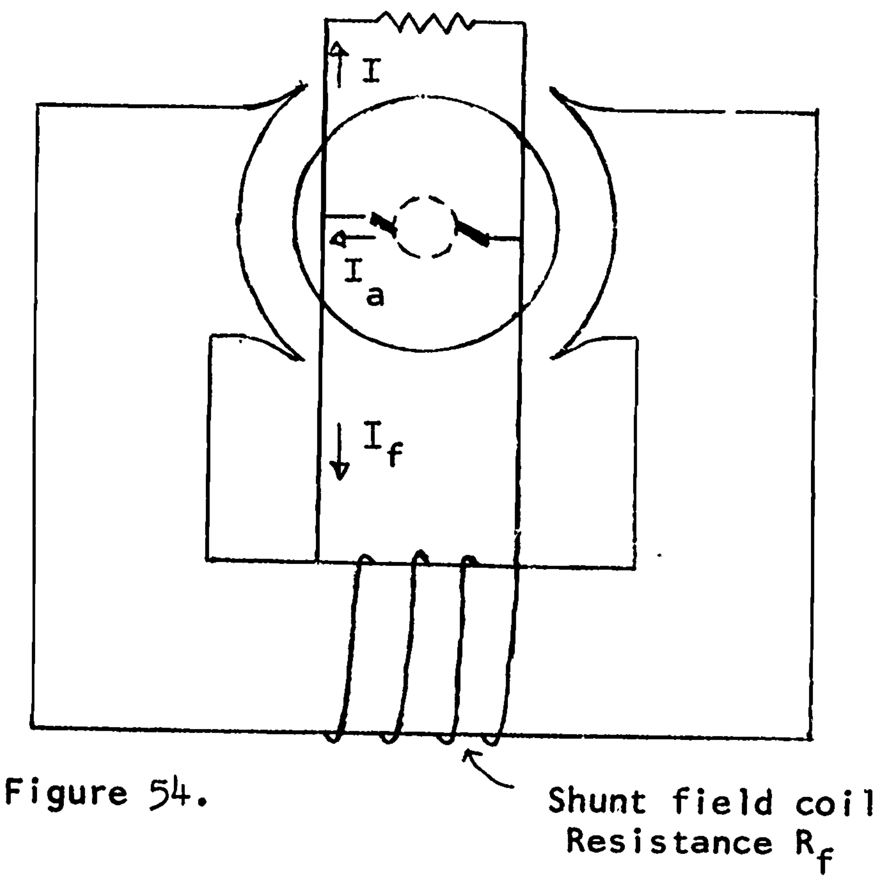


Figure 54.

A-c generators must get their d-c field current from a separate source, and hence when discussing the equivalent circuit for an a-c machine it was assumed that the d-c field current remained constant. In contrast, a d-c machine can provide its own d-c field <u>excitation</u>. One way to do this is to use a <u>shunt</u> field coil, as shown in Fig. 54. The field coil has many turns so that the necessary ampere turns can be obtained with a small current I_f, and a high resistance in order to limit I_f to the required value. (By "small", we mean I_f is many times smaller than the maximum load current I.)

At constant speed, the emf E of the machine is proportional to the airgap flux and therefore is a function of I_f. Thus, when the terminal voltage V changes with load current, this will in turn affect E (since $I_f = V/R_f$). Thus, in the equation

$$V = E - I R_a$$

E will be a function of load current through the mechanisms of ____________________ ____________________ and varying ____________________________________.

Answers:

armature reaction

field current

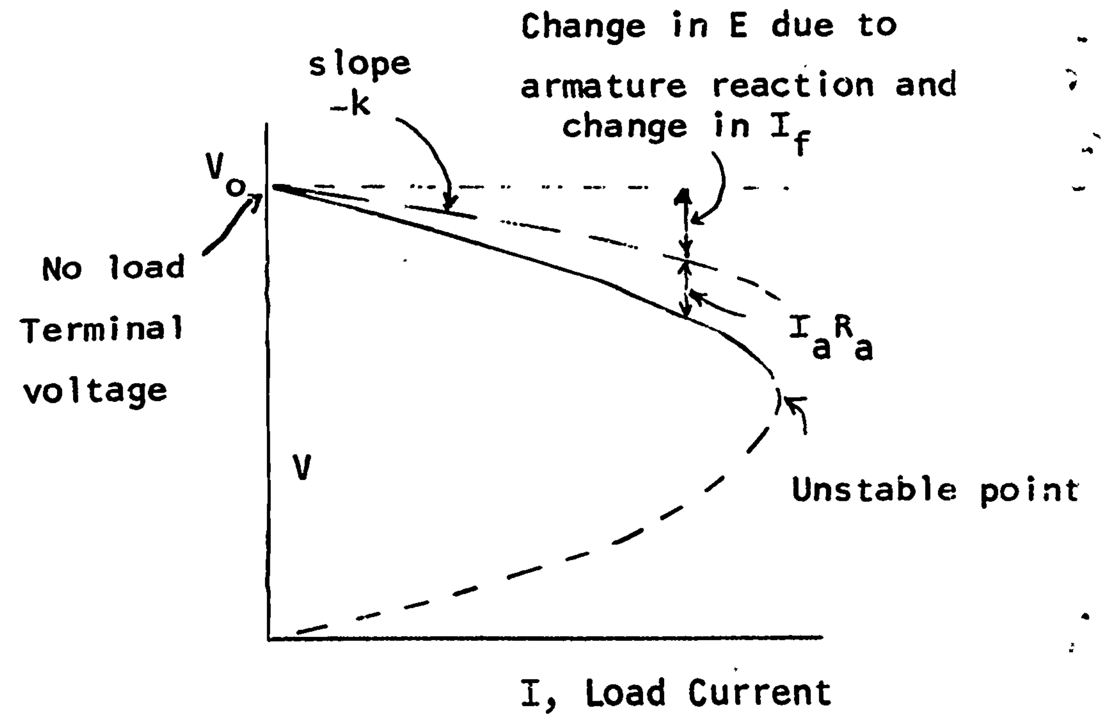
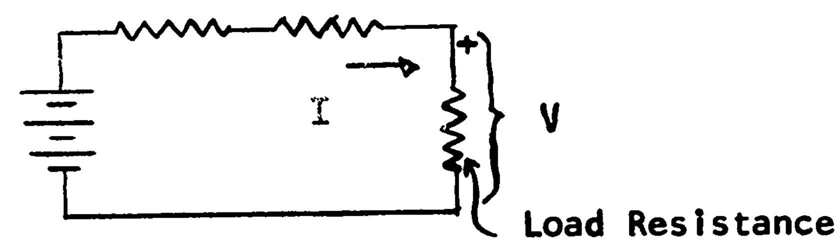


Figure 55.

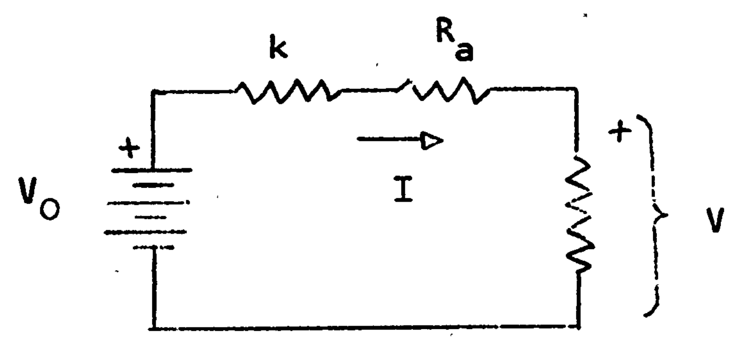
As a result of these phenomena, V will vary somewhat as shown in Fig. 55. If the load resistance is made too low, causing I to increase too far, an unstable point will be reached as indicated by the dotted curve, and the voltage will reduce to zero.

An approximate equivalent circuit can be used for the nearly linear portion of the V - I curve. The E curve is approximated by a straight line of slope = -k, as indicated. The equivalent circuit will then be like this



Label the (ideal) battery and two resistors in terms of quantities appearing on Fig. 55.

Answers:



Note: k and R_a might be combined into a single resistor. However, k is not associated with a power loss, and for that reason it is well to keep them separate.

There are other means of obtaining the field current of a d-c generator from its own output. Another one is to use a coil of low resistance and a few turns connected in <u>series</u> with the load (a series field coil). Such a machine has very different characteristics, because E is almost proportional to the load current I. In fact, for part of its operating range such a generator approximates a <u>current source</u>.

In addition, it is possible to obtain a wide variety of characteristics by combining shunt and series field coils, as in a <u>compound</u> machine. In fact, by having the ampere turns of the series coil <u>aid</u> those of the shunt field, it is possible to make V <u>rise</u> as I increases.

A detailed consideration of these various types of excitation is a topic in the study of machinery, and will not be undertaken here.

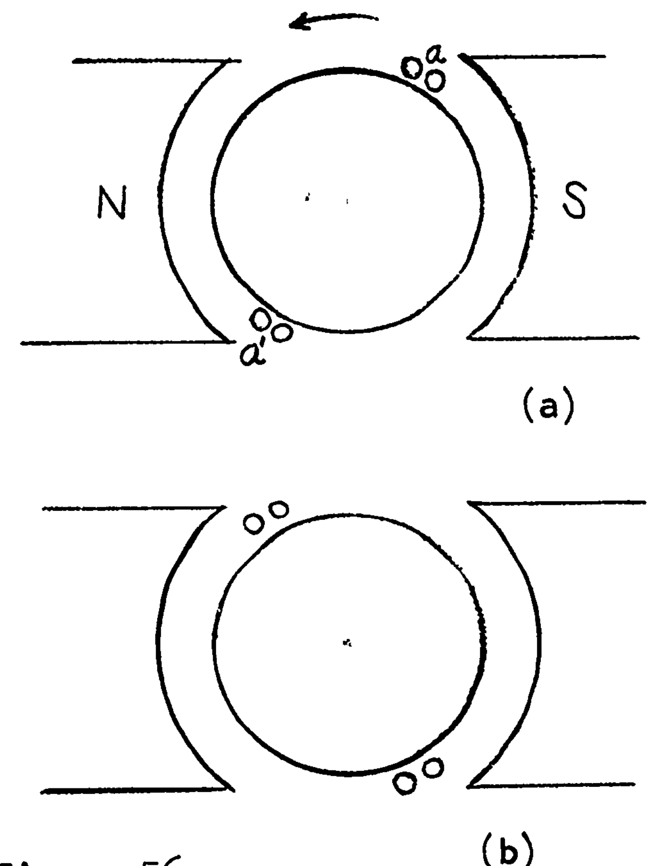


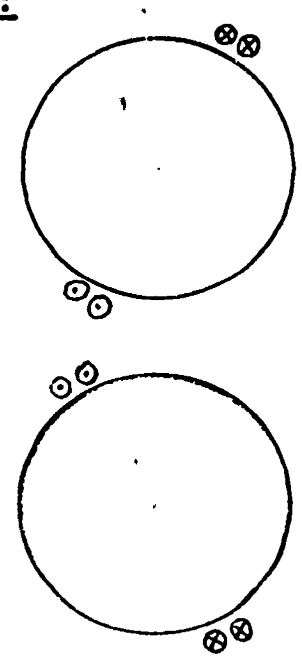
Figure 56.

Improving Commutation

One more topic of practical importance in d-c machines is to be considered. Refer to Fig. 56a which shows the two loops (a-a') in the same position as in Fig. 52. Figure 56b shows them 1/6 revolution later. Place dots and crosses in each circle, to represent current directions consistent with the direction of the airgap flux and the direction of rotation of Fig. 52.

You can conclude that in undergoing this amount of rotation the direction of current [illegible] in the pair of coils labeled a-a'.

Answers:



reverses.

In fact, the reversal takes place in a much smaller angle of rotation, occurring in the small interval required for the brush to pass from one segment to the next.

Since each coil has an inductance L, there will be an emf,

$$-L\frac{di}{dt}$$

induced by virtue of this current reversal. For example, if the current in the coil is 10 amps. and it reverses in .001 sec., the average value of di/dt will be ______________.

This emf can be troublesome, causing sparking as the brush passes from one segment to the next.

Answer:

20,000

The change in current is

$10 - (-10) = 20$ amp.

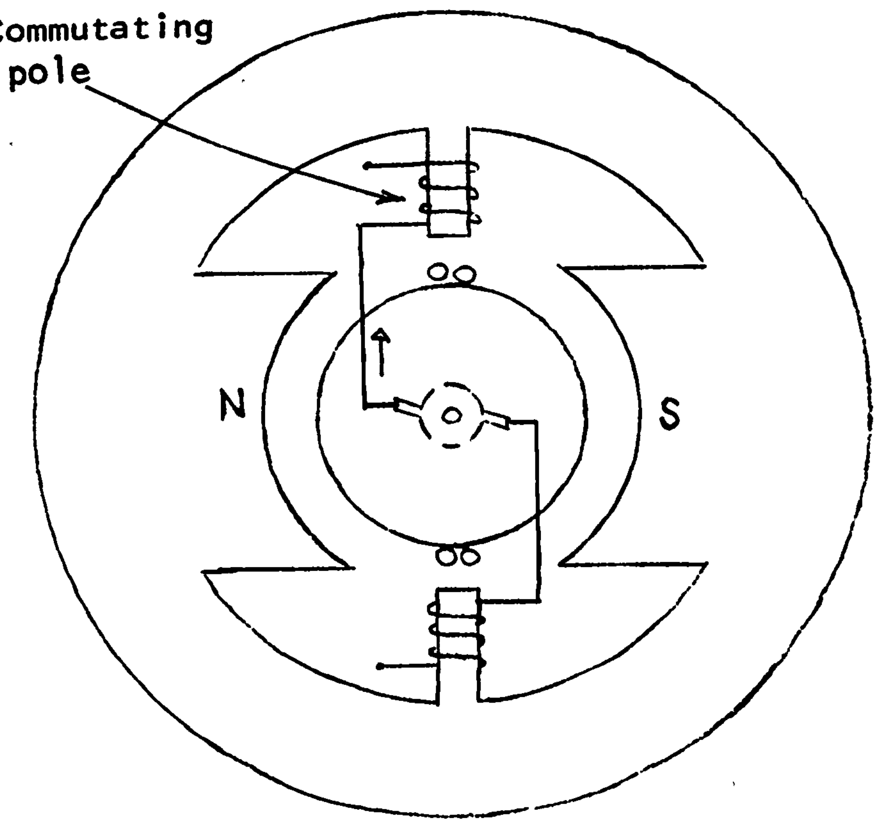


Figure 57.

The difficulty is overcome by the use of <u>interpoles</u>. There are small auxiliary magnetic poles placed as shown in Fig. 57. Each pole piece has a winding of a small number of turns which carries the <u>load</u> current. Thus, the strength of the interpole field is proportional to the amount of current being reversed in an armature coil.

From Fig. 57 it is evident that the coils undergoing a current reversal are at the same time passing under the interpoles. By proper choice of the winding direction and number of turns, it is possible to have the emf induced by the interpoles nearly cancel the $L\,di/dt$ emf.

Will the effect of the interpoles (for a given load current) be the same at all speeds? ____________

<u>Answer:</u>

Yes.

Suppose the speed is doubled, but I is the same. Since the flux from the interpoles is determined by I, it will be the same. Then, the emf induced by the interpoles will double. However di/dt will also double, since the time of "switching" will be cut in half. Thus, both emfs will vary in proportion, and will still cancel.

Review Problem

A d-c generator is to be designed along the lines discussed in this text. The open circuit terminal voltage is to be 250 volts, with a maximum to minimum ripple of 3 volts. The speed and strength of the magnetic field is such that the maximum emf (peak value) in each conductor is 2.1 volts, and it may be assumed that this emf varies sinusoidally with angular position of the rotor.

Determine the total number of coils needed on the armature, and the number of turns per coil.

Note: Since the number of coils and number of turns must be integers, use the nearest integers to the numbers obtained in your calculations.